WP EN2019-09

Ability and limitations of optimization models for computing the equilibrium in competitive energy markets

K. Poncelet, S. Kaminski, E. Delarue

TME Working Paper - Energy and Environment
Last update: July 2019 2016

KU Leuven Energy Institute
TME Branch

Ability and limitations of optimization models for computing the equilibrium in competitive energy markets

Kris Poncelet[%,+], Steffen Kaminski[%,+], and Erik Delarue[%,+]
[%]University of Leuven (KU Leuven), Energy Institute, Celestijnenlaan 300, B-3001 Leuven, Belgium
[+]EnergyVille, Thor Park, B-3600 Genk, Belgium

Abstract

The majority of energy-system or power-system planning models relies on optimization to compute the long-run or intertemporal market equilibrium. In deregulated energy/electricity markets, investments in most energy assets are made by private companies, aiming to maximize profits. In this context, the decisions made by these companies depend on their behavioral characteristics (e.g., strategic behavior, risk-averse behavior) as well as the market design and policies in place. However, optimization models face certain restrictions in representing some of these elements. This paper aims to provide insights in the ability and the limitations of optimization models for computing market equilibria. To this end, we analyze the relation between optimization and equilibrium models and derive a framework which allows modelers to assess whether a given equilibrium problem can be cast into an optimization model. We then apply this framework to illustrate the inherent limitations of optimization models for examples of equilibrium problems, which are relevant in the context of investment planning.

1 Introduction

1.1 The Changing Role of Long-term Energy-system Planning Models

The first applications of mathematical long-term power-system and energy-system planning models have taken place before the liberalization of the European energy/electricity markets. In that context, a central planner (e.g., a government or a state-owned or regulated utility) faced the problem of determining a long-term investment plan that minimizes the total cost of the energy provision. As such, long-term planning models, formulated as optimization problems, were developed.

In the context of a broader wave of deregulation of the Western economies, economists argued that competitive energy/electricity markets would provide better incentives for making efficient capacity expansion and operational decisions, which should ultimately lead to lower prices for consumers. For these reasons, during the 1980s-1990s, a gradual process of liberalization of the energy system (e.g., electricity and gas markets) has started (1). In essence, this process involved the unbundling of generation and retail activities from transmission and distribution activities, the introduction of competitive markets for generation and supply (retail) of electricity and gas, the introduction of natural monopolies for transmission and distribution, and the introduction of an independent regulator in charge of monitoring both the market-activities (generation and supply) and the regulated activities (transmission and distribution) (2, 3). Since the liberalization and deregulation of the energy markets, infrastructure investment and operational decisions are made by private companies aiming to maximize their profits. Hence, there is no longer a central authority that plans the energy system in order to maximize welfare.

As a result of the deregulation, the role of long-term planning models has changed from determining the optimal investment planning for direct execution to steering the market outcome in the desired direction (4). In this regard, long-term energy-system planning models form valuable tools for policy making. Typically, these models are used to analyze and compare a number of distinct scenarios, i.e., possible transition pathways of the considered energy system. Depending on the question that needs to be addressed, different types of scenario exercises can be performed. One can distinguish between normative/prescriptive scenarios and descriptive scenarios.

In normative or prescriptive scenarios, one is typically interested in determining the optimal transition pathway of the energy system towards achieving a desired future state. A typical example is when one would like to achieve a certain share of renewable energy sources (RES) or a certain reduction of greenhouse gas emissions within a certain timeframe, and the questions that one wants to answer have a normative character, e.g., which energy sectors should decarbonize first, which technologies are essential for achieving the target cost effectively, etc. As such, normative scenarios provide information about the ideal transition of an energy system towards the stated objective (how do we want the energy system to evolve?) (5, 6).

Descriptive scenarios, on the other hand, do not impose a desired future state, but rather aim to describe a likely evolution of the energy system, i.e., how do we expect the energy system to evolve given certain assumptions on fuel prices, technology cost evolutions, policy interventions and market design. Such scenarios can be used for instance to evaluate whether certain policy measures or a market design could achieve the desired state, and if so, under which conditions (7). For instance, policy-makers could decide to implement a subsidy scheme for solar photovoltaic (PV) panels and wind turbines with the idea of reaching a certain target for the reduction of greenhouse gas emissions, but without imposing the target itself. Descriptive scenario could then be used to assess the adequacy of the planned subsidy scheme and the resulting environmental, social and economic implications. Such descriptive scenarios are therefore crucial for translating the visions (how do we want the energy system to evolve?) that can be created using normative scenarios to a specific policy portfolio (how will we make sure that the desired transition will effectively be realized?).

In recent years, multiple studies have developed and analyzed scenarios for the evolution towards a sustainable energy/electricity system, either focusing on the feasibility and implications of realizing ambitious targets for renewable energy or the reduction of greenhouse gas (GHG) emissions (e.g., (8, 9, 10)), the role of specific technologies (e.g., (11, 12, 13)), the role of policy instruments (e.g., (14, 15)) or the role of the market design (e.g., (16, 17, 18, 19, 20, 21). In addition to such academic studies, planning models have been regularly deployed for providing direct policy support. In Europe, the PRIMES model (22) in particular has been used frequently for developing European Union (EU) policy (23). In the United States (US), the NEMS model of the Energy Information Agency (EIA) of the US Department of Energy has been used regularly for underpinning energy policy. Among others, this model has been

used to analyze the impact of the proposed American Clean Energy and Security Act of 2009, and is used to generate the annual energy outlook (AEO) of the US (24). Other popular examples of long-term energy-system planning models used for policy support are MARKAL/TIMES (25, 26, 27) (see e.g., (28, 23, 29)), MESSAGE (30). In addition, certain models focus in detail on the evolution of the electrical power system. Well known examples are the ReEDs model (31), the LIMES model (32) and the Resource Planning Model (33).

1.2 Mathematical Models Used to Generate Descriptive Scenarios

In this paper, our focus is on the models used to generate descriptive scenarios. As stated above, the aim of these scenarios is to describe the likely evolution of the energy/electricity system given that certain policies and market designs are in place. To determine this likely evolution, many models aim to compute the long-run or intertemporal market equilibrium.

Optimization models are a natural fit for generating normative scenarios. By maximizing welfare, the cost-effective transition pathway from a societal perspective can be analyzed. However, optimization models can and are also used to analyze the long-run or intertemporal market equilibrium. In that regard, optimization models rely on the fact that the total surplus is maximized in the equilibrium found in competitive markets where different economic agents aim to maximize their own profit (relating to the famous invisible hand of Adam Smith) (26, 34, 35, 36). Thus, by maximizing the total surplus, the competitive equilibrium can be computed. As stated by (26), this allows to "shift the model's rationale from a global, societal one (social welfare maximization) to a decentralized one (individual utility maximization)".

Optimization models have the main advantage that they can rely on fast and efficient solvers to study model instances with the required large geographical, temporal and sectoral scope (35, 37, 38). Recently, this has become increasingly relevant because a higher level of temporal, technical and spatial detail is required to properly account for the challenges related to the integration of intermittent renewables (39, 40). As a result, optimization models have remained the most popular tools for long-term energy-system and power-system planning (41).

However, optimization models face a number of limitations in terms of the conditions for which they can compute the market equilibrium. A first limitation is that optimization models implicitly assume price-taking agents. As a result, an optimization model cannot be used to determine the equilibrium if certain agents behave strategically (e.g., oligopoly situations). A second limitation of optimization models is that they rely on maximizing the total surplus by integrating the area between the supply and demand curve. In certain cases where demand and/or supply functions are represented by analytical expressions (e.g., derived from econometric studies), the demand or supply function might not be integrable[1]. In these cases, an optimization model cannot compute the equilibrium. A final limitation of optimization models is optimization models cannot compute the equilibrium if certain policies (e.g., a VAT tax) or market designs (e.g., minimum prices) are in place, or if the agents are assumed to have certain behavioral characteristics (e.g., risk-averseness).

A variety of techniques has been used to compute the market equilibrium for those instances where an optimization model cannot be used. A first technique that has been used regularly is to use an iterative procedure in which an optimization model is solved and modified recursively until the market equilibrium is found (Gauss-Seidel type of algorithms) (36). Greenberg and Murphy have for instance showed that such an algorithm can be used to solve equilibrium problems in the presence of price regulations such as tax programs, average cost pricing and price ceilings (42). The same approach has more recently been used by (43) to analyze the impact of feed-in tariffs for renewables. Lately, it has become more common to formulate equilibrium problems as mixed complementarity problems (MCPs), which, due to advancements in solver algorithms, can now be solved reasonably fast (36, 44, 37). For instance, (45) used an MCP to determine the long-run equilibrium under different emission allowance allocation rules. (46) and (19) have formulated an MCP to compare energy-only and capacity market organizations when investors are risk-averse. Another example can be found in (47) who developed an MCP for analyzing the impact of introducing average cost pricing for a consortium of large industrial consumers. For simulating the impact of certain types of strategic behavior, such as Stackelberg games, in which a strategic player can anticipate the reaction of a number of followers, the equilibrium problem is typically formulated as a bilevel problem. If there is a single leader, the equilibrium is typically formulated as a mathematical problem with equilibrium constraints (MPEC). If there is competition between multiple leaders, the equilibrium is typically formulated as a equilibrium problem with equilibrium constraints (EPEC) (see

e.g., (48, 49, 50, 51) for examples analyzing the impact of different degrees of market power on the long-run equilibrium). For a detailed description of these mathematical techniques, we refer to (44), and (37).

1.3 Goal and Contribution

As discussed in the previous section, there are a number of conditions for which optimization models cannot compute the market equilibrium. The fact that optimization models implicitly assume perfectly competitive (i.e., price-taking) agents is well known, and can easily be communicated in non-mathematical terms. In contrast, the exact limitations of optimization models in representing specific market designs, policy instruments or other behavioral characteristics are less well known, and cannot easily be described in non-mathematical terms. In the literature, these limitations are typically mentioned on a case-by-case basis when such a limitation is encountered for the specific problem at hand (see the different examples of problems that were solved using Gauss-Seidel type of iterative algorithms or MCPs above). To the best of our knowledge, there is no general overview of the limitations of optimization models, and the corresponding implications for determining the equilibrium in deregulated competitive but imperfect energy markets. One exception is the recent paper of (36) that explains the limitations of optimization models for computing the market equilibrium in a non-mathematical manner, and provides a list of policy interventions that cannot be cast in an optimization model. However, the goal of their paper is to introduce MCPs and the benefits these could have for energy-system models, rather than to present an exhaustive overview of the limitations of optimization models. In addition, because a non-mathematical approach is taken, it is difficult to generalize the presented examples of the limitations of optimization models to other problems.

The goal of this paper is to present an overview of the possibilities and limitations of optimization models for computing the market equilibrium when specific policies or market designs are in place, and for different assumptions regarding the agents' decision-making behavior. This paper complements the literature (e.g., the recent work of (36)) by presenting the underlying mathematical reasons for these limitations of optimization models and provide a framework which allows assessing whether a given equilibrium problem could be cast into an optimization model. We restrict ourselves to imperfect energy markets in which none of the agents behaves strategically, i.e., all agents are assumed to be price takers. For an overview of different techniques for solving equilibrium problems where agents behave strategically, we refer to (37).

This paper targets people of the energy-system and power-system modeling community who want to gain insights into how optimization models represent market equilibria, what the limitations of optimization models are to represent market equilibria and what other techniques could be used in such cases. Increasing awareness of the inherent limitations of optimization models to solve equilibrium problems is essential for deciding on a long-term strategy for the type of model to develop.

1.4 Outline

The remainder of this chapter is organized as follows. First, Section 2 analyzes the relation between optimization problems and equilibrium problems and presents a framework to determine whether an equilibrium problem can be cast into a system optimization model. This framework highlights a number of limitations of optimization models. These limitations are subsequently illustrated in Section 3, which presents topical examples of equilibrium problems containing policies, market designs and behavioral characteristics that are relevant in the context of planning in deregulated electricity markets but cannot be cast into a system optimization models. Finally, Section 4 summarizes and presents the main conclusions.

2 Limitations and Possibilities for Determining the Market Equilibrium Using Optimization Models

The methodology used in this paper to show the possibilities and limitations of system optimization models to compute market equilibria relies on the fact that (i) mixed complementarity problems (MCPs) form a natural way to describe market equilibria (36, 52), and (ii) MCPs generalize the class of linear and convex non-linear optimization problems with continuous variables. The latter implies that every linear or convex non-linear optimization problem with continuous variables can be converted to an MCP, but

the opposite does not hold (44). Hence, if a certain equilibrium problem can be formulated as an MCP, and this MCP cannot be derived from converting a system optimization problem into its equivalent MCP, we can conclude that this equilibrium cannot be computed directly by solving a system optimization model [2]. We restrict ourselves in this work to equilibrium problems with price-taking agents that can be formulated as an MCP[3].

By analyzing how equilibrium problem can be described as an MCP, and how a system optimization model can be cast into its equivalent MCP, we will derive certain conditions that need to be fulfilled in order for a system optimization problem to be able to compute the equilibrium. We will then interpret these mathematical conditions from an economic/market perspective, and provide a framework allowing modelers to easily check whether an equilibrium problem can be cast into an equivalent system optimization model, and to determine how the corresponding objective function of the system optimization model should look like.

The remainder of this sections is as follows: first Section 2.1 illustrates for a simple example of a generation expansion planning problem how the equilibrium problem can be formulated as an MCP and how a system optimization problem can be converted to an MCP. Next, Section 2.2 analyzes which conditions must be fulfilled in order for an equilibrium problem to be cast into an system optimization problem and the corresponding limitations for the use of optimization models for solving equilibrium problems. Finally, Section 2.3 briefly highlights the equilibrium problems that can be solved using optimization models.

2.1 Formulation of a Generation Expansion Planning Problem

2.1.1 Equilibrium problem.

Consider multiple price-taking generation companies (GenCos) i participating in a wholesale market. To simplify the problem, assume that each GenCo has the option to invest in generation capacity cap_i of a single technology, which is characterized by an annualized investment cost C_i^{CAPEX} and a constant generation cost C_i^{OPEX}. This generation capacity can be used to generate a power output $gen_{i,t}$ during every time step t (having a duration Δ_t) within the year. The generated electric energy can be sold in the market at a price p_t^{el}. The demand side in each time step t is represented by a given inverse demand function $f_{d,t}^{-1}(q_t)$. Here, the equilibrium problem is to find the long-run equilibrium.

In this example, each GenCo i faces the problem of determining the investment and operational decisions that maximize its long-run profits subject to (s.t.) certain constraints[4]:

$$\max_{cap_i, gen_{i,t}} \quad \sum_t (gen_{i,t}\Delta_t p_t^{el}) - cap_i C_i^{CAPEX} - \sum_t \left(gen_{i,t} C_i^{OPEX} \Delta_t\right) \tag{1a}$$

$$s.t. \quad cap_i - gen_{i,t} \geq 0 \quad (\gamma_{i,t}) \quad \forall t \tag{1b}$$

$$gen_{i,t} \geq 0 \quad \forall t \tag{1c}$$

$$cap_i \geq 0 \tag{1d}$$

In addition, the consumers aim to maximize their consumer surplus:

$$\max_{q_t} \quad \sum_t \left(\int_0^{q_t} f_{d,t}^{-1}(q_t') dq_t' \Delta_t \right) - \sum_t (q_t \Delta_t p_t^{el}) \tag{2a}$$

$$s.t. \quad q_t \geq 0 \quad \forall t \tag{2b}$$

Finally, the linking constraints need to be considered. These linking constraints are constraints that link together the variables of the different optimization problems (and thus the decision variables of the different agents). Typically, these are constraints that ensure that there is a balance between supply and demand, and are therefore sometimes referred to as market clearing constraints. Aside from ensuring a balance between the demand and supply of certain commodities, linking constraints are also frequently used to reflect the scarcity of certain commodities or reflect policy constraints that cap the total consumption/production of certain commodities. In this example, we only consider the balance between the supply and demand of electricity:

$$\sum_i gen_{i,t}\Delta_t = q_t \Delta_t \quad \forall t \tag{3}$$

2.1.2 MCP Formulation.

To derive the MCP formulation of the equilibrium problem, the Karush-Kuhn-Tucker (KKT) conditions of each optimization problem need to be determined. These KKT conditions are a set of equations and inequalities that form a mathematical expression of the necessary conditions for the optimal solution of an optimization problem. Under certain conditions, these KKT conditions are also sufficient. For instance, for linear and convex quadratic optimization problems, the KKT conditions are both necessary and sufficient. This means that any solution that satisfies the KKT conditions is effectively an optimal solution of the optimization problem[5] (44). For more information on deriving the KKT conditions, we refer to Chapter 4 of (53). By combining the KKT conditions of all optimization problems and adding the linking constraints, the MCP formulation of the equilibrium problem is derived. This MCP formulation of the equilibrium problem is thus nothing else than a set of equations and inequalities that must be fulfilled in the equilibrium. These equations and inequalities consist of conditions that must be satisfied in order for the solution to reflect the optimal decision making of the agents involved, conditions that reflect the constraints faced by each agent, and the linking constraints. Such an MCP can be solved using commercial solvers, such as the PATH solver (44).

The MCP of the generation expansion problem is shown below. Here, we make use of the perpendicular operator $\perp$. The presence of the perpendicular operator between two inequalities $g(x) \leq 0$ and $\alpha \geq 0$ represents an additional equation stating that at least one of the inequalities should be an equality, i.e., $g(x)\alpha = 0$.

$$p_t^{el}\Delta_t \leq C_i^{OPEX}\Delta_t + \gamma_{i,t} \quad \perp \quad gen_{i,t} \geq 0 \qquad \forall i, t \tag{4a}$$

$$\sum_t (\gamma_{i,t}) \leq C_i^{CAPEX} \quad \perp \quad cap_i \geq 0 \qquad \forall i \tag{4b}$$

$$cap_i - gen_{i,t} \geq 0 \quad \perp \quad \gamma_{i,t} \geq 0 \qquad \forall i, t \tag{4c}$$

$$f_{d,t}^{-1}(q_t) \leq p_t^{el} \quad \perp \quad q_t \geq 0 \qquad \forall t \tag{4d}$$

$$\sum_i gen_{i,t}\Delta_t = q_t\Delta_t \qquad \forall t \tag{4e}$$

Eq. (4a)-(4c) represent the KKT conditions of the GenCos optimization problems, Eq. (4d) is the KKT condition for the consumer and Eq. (4e) is a linking constraint that enforces a balance in the generation and consumption of the commodity electricity.

From Eq. (4a), we can derive that when a certain agent i decides to generate electricity using a certain technology, the price should be at least as high as the generation cost of that technology. If this is not the case, the generator will decide not to generate electricity. Moreover, from Eq. (4c) and Eq. (4a), we can deduce that if a technology is generating electricity, but less than its installed capacity, this plant clears the market, and hence, the price equals the generation cost of that technology[6]. When a technology is generating at maximal capacity, the price can be higher than the generation cost of that technology, and the owners of the plants of that technology can earn infra-marginal rents (indicated by the dual variable $\gamma_{i,t}$). In terms of investments, we can see from Eq. (4b) that an agent will only invest in capacity of a certain technology if the infra-marginal rents that would be earned during the different time steps are sufficient to cover the investment costs. Moreover, when an agent invests in a certain technology, it will do so up to the point where the infra-marginal rents are just sufficient to cover the investment costs. From Eq. (4d), it follows that the consumers consume up to the point where the inverse demand function, i.e., their willingness to pay, equals the electricity price.

2.1.3 Optimization Problem Formulation.

Considering the same GenCos and consumers, the solution yielding maximal total surplus is the solution to the following optimization problem:

$$\max_{q_t, cap_i, gen_{i,t}} \quad \sum_t \Big(\int_0^{q_t} f_{d,t}^{-1}(q'_t) dq'_t \Delta_t \Big) - \sum_i cap_i C_i^{CAPEX} - \sum_i \sum_t \big(gen_{i,t} C_i^{OPEX} \Delta_t \big) \tag{5a}$$

$$s.t. \quad cap_i - gen_{i,t} \geq 0 \quad (\gamma_{i,t}) \quad \forall i, t \tag{5b}$$

$$gen_{i,t} \geq 0 \quad \forall i, t \tag{5c}$$

$$cap_i \geq 0 \quad \forall i \tag{5d}$$

$$q_t \geq 0 \quad \forall t \tag{5e}$$

$$\sum_i gen_{i,t} \Delta_t = q_t \Delta_t \quad (\lambda_t) \quad \forall t \tag{5f}$$

This optimization problem can be solved directly using efficient optimization solvers. However, to illustrate the equivalence between the solution of the optimization problem (5) and the MCP formulation of the equilibrium problem (4), we will convert the above optimization problem (5) to an MCP via its KKT conditions. Note that in this case, there is only a single optimization problem and therefore the linking constraints are internal constraints of this optimization problem. Hence, the MCP formulation of the surplus maximization problem is simply the set of KKT conditions of the surplus maximization problem, i.e.:

$$\lambda_t \Delta_t \leq C_i^{OPEX} \Delta_t + \gamma_{i,t} \quad \perp \quad gen_{i,t} \geq 0 \qquad \forall i, t \tag{6a}$$

$$\sum_t (\gamma_{i,t}) \leq C_i^{CAPEX} \quad \perp \quad cap_i \geq 0 \qquad \forall i \tag{6b}$$

$$cap_i - gen_{i,t} \geq 0 \quad \perp \quad \gamma_{i,t} \geq 0 \qquad \forall i, t \tag{6c}$$

$$f_{d,t}^{-1}(q_t) \leq \lambda_t \quad \perp \quad q_t \geq 0 \qquad \forall t \tag{6d}$$

$$\sum_i gen_{i,t} \Delta_t = q_t \Delta_t \qquad \forall t \tag{6e}$$

For the presented example, by noting that the dual variable of the market clearing condition (Eq. (5f)) of the optimization problem represents the equilibrium price (i.e., $\lambda_t = p_t^{el}$), it becomes clear that the MCPs (6) and (4) are equivalent. Hence, instead of having to solve the MCP, a faster computation of the equilibrium is possible by simply solving the optimization problem (problem (5)) (44, 38, 36). Both approaches to solving the equilibrium problem are schematically represented in Fig. 1.

As stated earlier, any linear or convex non-linear optimization problem with continuous variables (that satisfy certain constraint qualifications) can be converted to an MCP via its KKT conditions, but it will not always be possible to formulate an optimization problem of which the optimal solution represents the equilibrium (44, 36). More specifically, when we stated earlier that an optimization model cannot be used to compute the equilibrium in certain circumstances, we mean that it is not possible to formulate an optimization problem such that the KKT conditions of this optimization problem correspond to the MCP formulation of the equilibrium problem[7].

2.1.4 A Closer Look at the Equivalence.

To gain insights into the limitations of optimization models, it is relevant to have a further look at how the surplus maximization problem (problem (5)), via its KKT conditions, leads to the same MCP as the MCP formulation of the equilibrium problem.

In this regard, it is of interest to observe that in the presented generation expansion problem, the KKT conditions of the surplus maximization problem comprise the KKT conditions of each individual agent's optimization problem. The KKT conditions of each agent's optimization problem reflect both the constraints faced by each agent and the conditions for the optimal decision making of each agent. The conditions for the optimal decision making in turn consist of three types of terms: costs/revenue terms related to participation in the markets for which the prices are endogenously determined (i.e., the endogenous markets), terms related to exogenously specified costs/revenues and terms related to the shadow prices of the agent's constraints. Applied to the KKT conditions of the GenCo in the generation

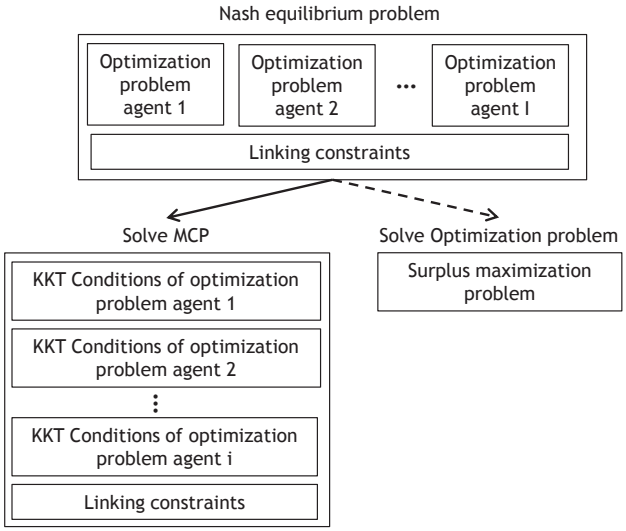

Figure 1: Schematic of different approaches to solving a Nash equilibrium problem. The dashed arrow indicates that not all equilibrium problems that can be formulated as a mixed complementarity problem (MCP) can be solved by solving a single optimization model.

expansion planning problem (Eq. (4a)-(4c)), Eq. (4c) reflects the constraint faced by the agent and Eq. (4a)-(4b) reflect the conditions for optimally deciding on the power generation in each time step and the installed capacity respectively. In these conditions for optimal decision making, the term $p_t^{el}\Delta_t$ reflects the revenues from participating in the electricity market, the terms $C_i^{OPEX}\Delta_t$ and C_i^{CAPEX} are exogenously specified operational and investment costs, and $\gamma_{i,t}$ are the infra-marginal rents related to the capacity constraint.

As shown in the example above, a surplus maximization problem directly integrates both the agents' constraints[8] (Eq. (5b)-(5e)) and the exogenously specified cost/revenue components (the terms $cap_i C_i^{CAPEX} + \sum_t (gen_{i,t} C_i^{OPEX} \Delta_t)$ in Eq. (5a)). Hence, the corresponding terms will appear identically in the KKT conditions of the surplus maximization problem as in the KKT conditions of the agent's optimization problem.

The main difference relates to the terms reflecting the revenues/costs from the participation in the endogenous markets. These revenue/cost terms related to the endogenous markets are explicitly represented in the objective function of each agent's optimization problem (see e.g., the term $\sum_t (gen_{i,t} p_t^{el} \Delta_t)$ in Eq. (1)) and hence appear in the KKT conditions of the agents optimization problem. In contrast, in the surplus maximization problem (5), no revenue or cost terms related to endogenous markets are specified. Nevertheless, the KKT conditions of the surplus maximization problem also contain these terms. This is because the cost or revenue terms related to the endogenous markets now appear indirectly in the KKT conditions of the surplus maximization problem via the linking constraints.

Each linking constraint integrated in a surplus maximization problem will thus indirectly describe a market, i.e., both the price (dual variable of the linking constraint) and the variables receiving/having to pay this price are indirectly specified via the linking constraints. In the example above, the linking constraint ensuring a balance between the supply and demand of electricity indirectly specifies that every unit of electricity generated in time step t, i.e., $gen_{i,t}\Delta_t$, receives a payment λ_t. Similarly, every unit of electricity consumed in time step t requires a payment of λ_t.

The information presented above is summarized in Fig. 2 and Fig. 3, which schematically illustrate how the MCP formulation of an equilibrium problem and the MCP formulation of a cost minimization problem is formed respectively. In addition, Fig. 4 gives a mathematical overview of the generic structure of an optimization problem and an equilibrium problem and how both are cast to an MCP.

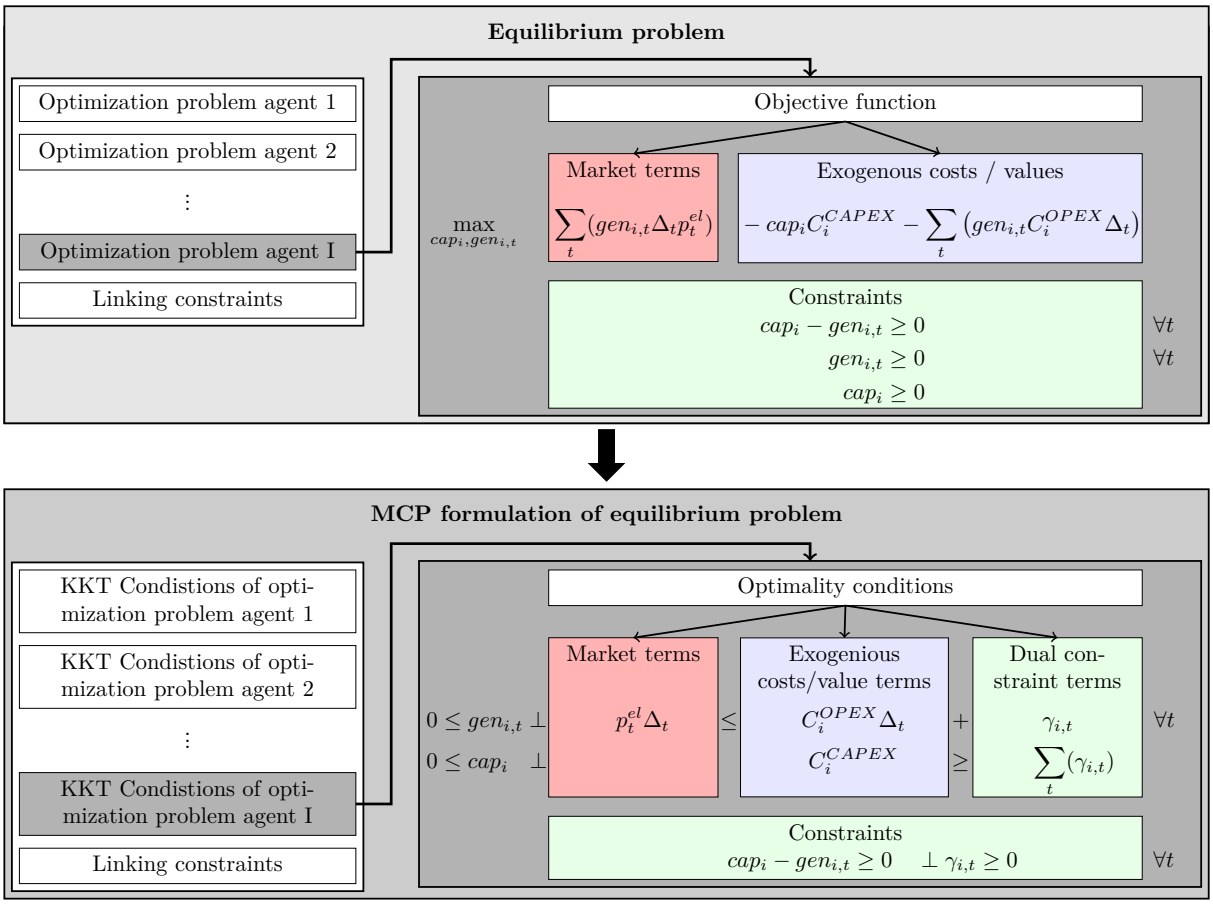

Figure 2: Schematic of how the mixed complementarity problem (MCP) formulation of an equilibrium problem is formed.

2.2 Limitations of Optimization Models

2.2.1 Inherent Assumptions Made in System Optimization Models

The main limitation of optimization problems is related to duality. In the previous section, we have shown that all revenues and cost terms related to the endogenous markets appear in the MCP derived from the surplus maximization problem indirectly via the linking constraints and the corresponding dual variables. These linking constraints thus not only represent certain physical or policy constraints of the optimization problem, but also specify the remuneration in the markets implicitly formed around each of these constraints. As such, optimization models cannot distinguish between a physical or policy constraint on the one hand and the revenues and costs attached to the variables appearing in this constraint via the market implicitly formed around this constraint on the other hand. This leads to three assumptions that are inherently made in optimization models and are listed below:

1. all agents' variables contributing to a certain linking constraint participate in a paid-as-cleared market implicitly formed around this linking constraint. This implicitly formed market provides a unique endogenously determined price (the dual variable of that linking constraint) that applies to all variables contributing to that linking constraint[9];

2. the endogenously determined market price does not directly influence the value of variables not appearing in the corresponding linking constraint;

3. all agents have the same valuation of the revenues or costs related to the participation in a certain market formed around a linking constraint.

In addition, optimization problems cannot contain dual variables in the primal problem formulation

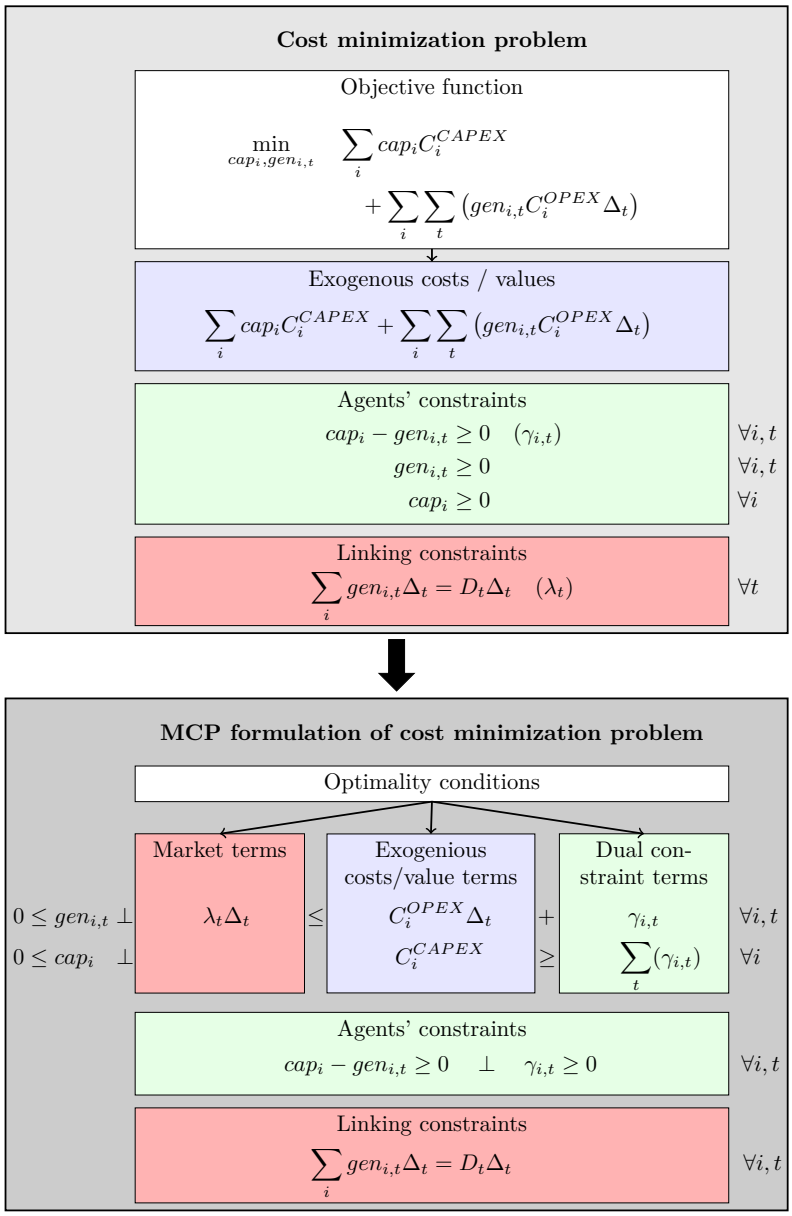

Figure 3: Schematic of how the mixed complementarity problem (MCP) formulation of a cost minimization problem is formed.

and can therefore not represent constraints of agents that contain endogenously determined market prices. This leads to a fourth assumption inherently made in optimization models:

4. the decision space of each agent is not dependent on the endogenously determined market prices[10].

If an equilibrium problem violates one of the above assumptions, the equilibrium problem cannot be cast into a system optimization model. These inherent assumptions hence restrict the use of optimization models for solving equilibrium problems.

2.2.2 Necessary Conditions.

The inherent assumptions adopted in optimization models, as discussed above, can be mathematically expressed via two necessary conditions for an equilibrium problem to be cast into an optimization model.

The first condition relates to the inherent assumptions 1-3.

Surplus maximization problem

$$Max_{\mathbf{x}} \quad F(\mathbf{x})$$

s.t.:

$$g_i^{n_i}(\mathbf{x}_i) \geq 0 \quad (\alpha_i^{n_i}) \quad \forall i = 1, \ldots, I \quad n_i = 1, \ldots, N_i$$

$$\mathbf{x} \geq 0$$

$$v^1(\mathbf{x}) \geq 0 \quad (\gamma^1)$$

$$\vdots$$

$$v^M(\mathbf{x}) \geq 0 \quad (\gamma^M)$$

Equilibrium problem

Optimization problem

Agent 1

$$Max_{\mathbf{x}_1} \quad \theta_1(\mathbf{x}_1, \boldsymbol{\gamma})$$

s.t.:

$$g_1^1(\mathbf{x}_1) \geq 0 \quad (\alpha_1^1)$$

$$\vdots$$

$$g_1^{N_1}(\mathbf{x}_1) \geq 0 \quad (\alpha_1^{N_1})$$

$$\mathbf{x}_1 \geq 0$$

$\ldots$

Optimization problem

Agent I

$$Max_{\mathbf{x}_I} \quad \theta_I(\mathbf{x}_I, \boldsymbol{\gamma})$$

s.t.:

$$g_I^1(\mathbf{x}_I) \geq 0 \quad (\alpha_I^1)$$

$$\vdots$$

$$g_I^{N_I}(\mathbf{x}_I) \geq 0 \quad (\alpha_I^{N_I})$$

$$\mathbf{x}_I \geq 0$$

$$v^1(\mathbf{x}) \geq 0 \quad \perp \quad \gamma^1 \geq 0$$

$$\vdots$$

$$v^M(\mathbf{x}) \geq 0 \quad \perp \quad \gamma^M \geq 0$$

MCP formulation of the surplus maximization problem

$$\frac{\partial F(\mathbf{x})}{\partial \mathbf{x}_1} + \sum_{n_1=1}^{N_1} \alpha_1^{n_1} \frac{\partial g_1^{n_1}(\mathbf{x}_1)}{\partial \mathbf{x}_1} + \sum_{m=1}^{M} \gamma^m \frac{\partial v^m(\mathbf{x})}{\partial \mathbf{x}_1} \leq 0 \quad \perp \quad \mathbf{x}_1 \geq 0$$

$$\vdots$$

$$\frac{\partial F(\mathbf{x})}{\partial \mathbf{x}_I} + \sum_{n_I=1}^{N_I} \alpha_I^{n_I} \frac{\partial g_I^{n_I}(\mathbf{x}_I)}{\partial \mathbf{x}_I} + \sum_{m=1}^{M} \gamma^m \frac{\partial v^m(\mathbf{x})}{\partial \mathbf{x}_I} \leq 0 \perp \quad \mathbf{x}_I \geq 0$$

$$g_i^{n_i}(\mathbf{x}_i) \geq 0 \quad \perp \quad \alpha_i^{n_i} \geq 0 \quad \forall i = 1, \ldots, I \quad n_i = 1, \ldots, N_i$$

$$v^1(\mathbf{x}) \geq 0 \quad \perp \quad \gamma^1 \geq 0$$

$$\vdots$$

$$v^M(\mathbf{x}) \geq 0 \quad \perp \quad \gamma^M \geq 0$$

MCP formulation of the equilibrium problem

$$\frac{\partial \theta_1(\mathbf{x}_1, \boldsymbol{\gamma})}{\partial \mathbf{x}_1} + \sum_{n_1=1}^{N_1} \alpha_1^{n_1} \frac{\partial g_1^{n_1}(\mathbf{x}_1)}{\partial \mathbf{x}_1} \leq 0 \quad \perp \quad \mathbf{x}_1 \geq 0$$

$$\vdots$$

$$\frac{\partial \theta_I(\mathbf{x}_I, \boldsymbol{\gamma})}{\partial \mathbf{x}_I} + \sum_{n_I=1}^{N_I} \alpha_I^{n_I} \frac{\partial g_I^{n_I}(\mathbf{x}_I)}{\partial \mathbf{x}_I} \leq 0 \quad \perp \quad \mathbf{x}_I \geq 0$$

$$g_i^{n_i}(\mathbf{x}_i) \geq 0 \quad \perp \quad \alpha_i^{n_i} \geq 0 \quad \forall i = 1, \ldots, I \quad n_i = 1, \ldots, N_i$$

$$v^1(\mathbf{x}) \geq 0 \quad \perp \quad \gamma^1 \geq 0$$

$$\vdots$$

$$v^M(\mathbf{x}) \geq 0 \quad \perp \quad \gamma^M \geq 0$$

Figure 4: Schematic of how a surplus maximization problem and an equilibrium problem are cast to a mixed complementarity problem (MCP). The equilibrium problem consists of I agents all facing their own optimization problem. The decision variables of agent i are represented by the vector $\mathbf{x}_i$. Assuming price-taking agents, the objective function θ_i of an agent i is dependent on the agents decision variables and the vector of prices $\boldsymbol{\gamma}$, which are the dual variables of the M linking constraints. Each agent i faces a number N_i of constraints. The objective function of the surplus maximization problem is dependent on the decision variables of all agents (denoted by $\mathbf{x}$).

Condition 1: A necessary condition for an equilibrium problem to be cast in a linear or non-linear optimization problem with continuous variables is that for each agent i in the equilibrium problem, the objective function $\theta_i(\mathbf{x}_i, \boldsymbol{\gamma})$ of its respective optimization problem can be formulated as:

$$\theta_i(\mathbf{x}_i, \boldsymbol{\gamma}) = F(\mathbf{x}) + \sum_{m=1}^{M} \gamma^m v^m(\mathbf{x}) + a_i(\mathbf{x}_{-i}, \boldsymbol{\gamma}) \quad \forall i, \tag{7}$$

where $\mathbf{x}_i$ and $\mathbf{x}_{-i}$ are respectively the decision variables of agent i and the decision variables of all agents except agent i. The decision variables of all agents are indicated by $\mathbf{x}$. Finally, $v^m(\mathbf{x})$ represents the mth linking constraints of the equilibrium problem with corresponding price γ^m.

If a function $F(\mathbf{x})$ and functions $a_i(\mathbf{x}_{-i}, \boldsymbol{\gamma})$ can be found such that the above condition is satisfied for all agents i, the function $F(\mathbf{x})$ is the objective function of the optimization problem of which the optimal solution represents the equilibrium.

This condition shows that in order for an equilibrium problem to be cast into a system optimization problem (or by extension, any non-linear optimization problem with continuous variables), the terms containing endogenously determined market prices in the objective function of each agent must correspond to a strict format that relates to the linking constraints.

Condition 1 can be derived by demanding that the MCP derived from the surplus maximization problem is identical as the MCP formulation of the equilibrium problem. By assuming that the surplus maximization problem reflects the constraints faced by each agent as well as the physical/policy constraints linking decision variables from multiple agents (these constraints must be incorporated in the surplus maximization problem to ensure that the optimal solution to this problem is feasible), and using the notation from Fig. 4, the following condition must be satisfied in order for both MCPs to be identical[11] (see Fig. 4):

$$\begin{aligned} &\frac{\partial F(\mathbf{x})}{\partial \mathbf{x}_i} + \sum_{n_i=1}^{N_i} \alpha_i^{n_i} \frac{\partial g_i^{n_i}(\mathbf{x}_i)}{\partial \mathbf{x}_i} + \sum_{m=1}^{M} \gamma^m \frac{\partial v^m(\mathbf{x})}{\partial \mathbf{x}_i} \\ &= \frac{\partial \theta_i(\mathbf{x}_i, \boldsymbol{\gamma})}{\partial \mathbf{x}_i} + \sum_{n_i=1}^{N_i} \alpha_i^{n_i} \frac{\partial g_i^{n_i}(\mathbf{x}_i)}{\partial \mathbf{x}_i} \quad \forall i. \end{aligned} \tag{8}$$

Here, the terms $\sum_{n_i=1}^{N_i} \alpha_i^{n_i} \frac{\partial g_i^{n_i}(\mathbf{x}_i)}{\partial \mathbf{x}_i}$ relate to the constraints faced by agent i and, since these constraints are represented in the agent's optimization problem as well as the surplus maximization problem, thus appear in both the KKT conditions of the surplus maximization problem and the KKT conditions of the optimization problem faced by each agent i. As a result, these terms can be eliminated from the above condition. Eq. (8) than reduces to:

$$\frac{\partial F(\mathbf{x})}{\partial \mathbf{x}_i} + \sum_{m=1}^{M} \gamma^m \frac{\partial v^m(\mathbf{x})}{\partial \mathbf{x}_i} = \frac{\partial \theta_i(\mathbf{x}_i, \boldsymbol{\gamma})}{\partial \mathbf{x}_i} \quad \forall i. \tag{9}$$

Here, $F(\mathbf{x})$ represents the objective function of the surplus maximization problem. Hence, the term $\frac{\partial F(\mathbf{x})}{\partial \mathbf{x}_i}$ appears in the KKT conditions of the surplus maximization problem via its objective function. In contrast, the terms $\sum_{m=1}^{M} \gamma^m \frac{\partial v^m(\mathbf{x})}{\partial \mathbf{x}_i}$ appear in the KKT conditions of the surplus maximization problem via the linking constraints. Here, γ^m represents the dual variable of linking constraint m and thus reflects the price of a certain implicitly created market. In the MCP of the equilibrium problem, the only remaining terms in the above equation directly follow from the agents' objective functions $\theta_i(\mathbf{x}_i, \boldsymbol{\gamma})$. The agents' objective functions typically directly comprise terms related to the market prices (γ). By integrating both sides of the equation over the decision variables $\mathbf{x}_i$, we finally get the first condition as presented above (Eq. (7)).

The fourth inherent assumption/limitation of optimization problems directly follows from the fact that endogenously determined prices are dual variables of the surplus maximization problem, and hence cannot appear in the primal problem formulation[12]. This is more formally presented in the following condition:

Condition 2: A necessary condition for an equilibrium problem to be cast in a linear or non-linear optimization model with continuous variables is that for each agent i in the equilibrium problem, the constraints $g_i^{n_i}$ faced by agent i are not a function of the endogenously determined market prices γ^m.

2.2.3 Framework for Determining the Existence of a System Optimization Problem Equivalent to the Equilibrium Problem.

The inherent assumptions adopted in system optimization models (as described in Section 2.2.1) as well as the necessary conditions for an equilibrium problem to be cast into a system optimization model (as described in Section 2.2.2) provide a framework to check whether a given equilibrium problem could be cast into a system optimization model.

This framework consists of the following steps:

1. Formulate the objective function of all agents involved;
2. Formulate the physical or policy-related constraints linking the decision variables of the different agents;
3. Check whether the inherent assumptions 1-3 hold/Check whether Condition 1 holds;
4. Formulate the constraints each agent faces;
5. Check whether inherent assumption 4/Check whether Conditon 2 holds.

It is important to note that this framework is applicable to equilibrium problems with price-taking agents and without analytical supply/demand functions that might cause issues related to integrability.

As an example, this framework is applied to the investment planning problem described in Section 2.1. As the optimization problems faced by the different agents (Eq. (1) and Eq. (2)), as well as the linking constraints (Eq. (3)) have already been defined, we here restrict ourselves to steps 3 and 5. In this problem, there is only a single linking constraint, which is of the form $\sum_i gen_{i,t}\Delta_t - q_t\Delta_t = 0$ and of which the dual variable represents the price of electricity p_t^{el}. By looking at the objective functions of the generation companies (Eq. (1a)) and the consumers (Eq. (2a)), we can observe that all variables appearing in the linking constraint indeed participate in a paid-as-cleared market, where the electrical energy generated/consumed is remunerated/charged via a unique market price (i.e., the objective function of the generators contain a term $gen_{i,t}\Delta_t p_t^{el}$, and the objective function of the consumer contains a term $-q_t\Delta_t p_t^{el}$). Thus, the equilibrium problem is in line with the first inherent assumption made in system optimization models. Furthermore, in the agents' objective functions, we can see that the market price for electricity does not impact the value of other variables aside from those appearing in the linking constraints, which is in line with the second inherent assumption. In addition, the equilibrium problem conforms to the third inherent assumption, since all agents have an identical valuation of the revenues or costs related to participation in the electricity market. Finally, we can see that the equilibrium problem is in line with the fourth inherent assumption made in optimization models, as the endogenously determined electricty does not appear in any of the agents' constraints.

More rigorously, we could also directly apply Condition 1 to derive the system optimization model equivalent to the equilibrium problem. This is illustrated in the Appendix.

However, certain market imperfection introduced via the market design or policy interventions will result in markets or agents that deviate from this necessary condition. This is for instance the case when the value of a certain variable is determined outside of the market but at the same time contributes to linking constraint representing a certain physical or political constraint. For example, a renewable generator receiving a fixed feed-in tariff receives the feed-in tariff rather than the market price for every unit of generated electricity. Nevertheless, the electrical power generated by this renewable generator must enter in the linking constraint since it contributes to meeting the physical constraint requiring a balance between demand and supply and impacts the market. Hence, not all variables appearing in the linking constraint are values as in a paid-as-cleared market.

Another example of a deviation from these assumptions occurs if certain variables are remunerated using certain market prices despite the fact that these variables do not contribute to the linking constraint corresponding to that market. Consider as an example a subsidy scheme where investors in renewable

generators receive a certain subsidy per unit of installed capacity that is dependent on the market price. In this example, the capacity variable gets some remuneration that is dependent on the electricity price even though the capacity variable does not enter in the linking constraint ensuring the balance between supply and demand of electricity. More detailed illustrations of such problems are presented in Section 3.

2.3 Opportunities for Optimization Models

Despite the above-mentioned limitations of optimization models for determining the market equilibrium, many markets and market distortions can be simulated using optimization models. A recent report developed for the European Commission presents an overview of current market distortions alongside the methodologies that can be used to model these distortions (54). It turns out that the majority of the distortions listed in this report can be easily modeled using optimization models.

A first thing to note is that one can easily introduce additional markets (of the paid-as-bid format) in a system optimization model by introducing additional linking constraints. As such, one could easily simulate markets for capacity, ancillary services, renewable energy or emission permits. More specifically, one needs to specify the upper and/or lower bounds for the gross or net consumption of certain commodities or services, and how different technologies contribute to meeting these bounds.

In addition, also certain market distortions can be simulated. For instance, one group of market distortions relate to having a non-level playing field due for instance the lack of market access for certain technologies, overly stringent eligibility criteria or product definitions. To model these distortions, one can easily adapt the variables that can contribute to meeting a certain linking constraint (and hence participate in the market) and/or the extent to which different variables can contribute. For example, if storage technologies are not allowed to provide operating reserves, one can simply exclude storage related variables in the linking constraint that imposes the balance between the provision and the requirement for operating reserves.

Moreover, also other distortions listed in the report on distortions in European electricity markets, such as price-caps and sub-optimal market coupling can be easily simulated using optimization models. In addition, some incomplete markets can be simulated. For example, by considering the electricity balance constraints only on zonal level, zonal pricing (and thus the lack of nodal price signals) can be modeled (although this might imply that the solution is not technically feasible and hence redispatch is required).

Finally, also common policy interventions such as volume based instruments (e.g., emission trading schemes or green certificate systems) as well as direct subsidies and taxes (as long as the subsidy or tax is not dependent on the endogenous prices) can be easily represented in optimization models.

As such, we would like to stress that it is a common misconception that optimization models compute the market equilibrium assuming perfect competition. This misconception does likely originate from the fact that system optimization models implicitly assume price-taking agents. However, note that to ensure perfect competition, several conditions must be satisfied, of which price-taking behavior is only one. Other conditions include, among others, the lack of externalities, no barriers to entry or exit and no government intervention. Some of these market imperfections can clearly be simulated using optimization models.

3 Illustrations

In this section, we illustrate the limitations of optimization models by providing three equilibrium problems related to investments planning that cannot be solved directly using a system optimization model. More specifically, we provide an illustration of a policy intervention, a market design and agent behavior that cannot be directly represented in a system optimization model.

The first illustration addresses the problem of determining the long-run equilibrium when a green certificate scheme is introduced with a guaranteed minimum price for green certificates. This illustration is handled in depth, and numerical results for simulations on a small test system are provided. The second illustration looks at the problem of determining the long-run equilibrium in generation expansion planning when residential consumers can invest in solar PV panels and net metering is applied. Finally, the last illustration addresses several issues related to representing the equilibrium if the agents face uncertainty.

3.1 Minimum Price for Green Certificates

3.1.1 Problem Formulation.

Consider an equilibrium problem where a green certificate scheme is introduced to incentivize investments in renewable electricity generation. In this scheme, suppliers need to hand in green certificates corresponding to a fraction FR of the electricity sold, and generators are provided R_g green certificates for every MWh of electrical energy generated by RES. Generators and suppliers can then trade these certificates in a market for green certificates (market price p^{GC}). In addition, in order to reduce uncertainties for investors in renewable electricity, generators are guaranteed a minimum price for their green certificates. More specifically, generators have the option to sell their certificates to the distribution system operator (DSO), which is obliged to buy these certificates at the guaranteed minimum price ($P_g^{GC,DSO}$). Note that this minimum price can differ for different technologies g. The DSO itself has no obligation to hand in certificates. Hence, the DSO will sell all certificates that he was obliged to acquire at the minimum price back to the market. Fig. 5 provides a schematic overview of the green-certificate trading scheme. Among others, such a support system has been implemented in different regions in Belgium.

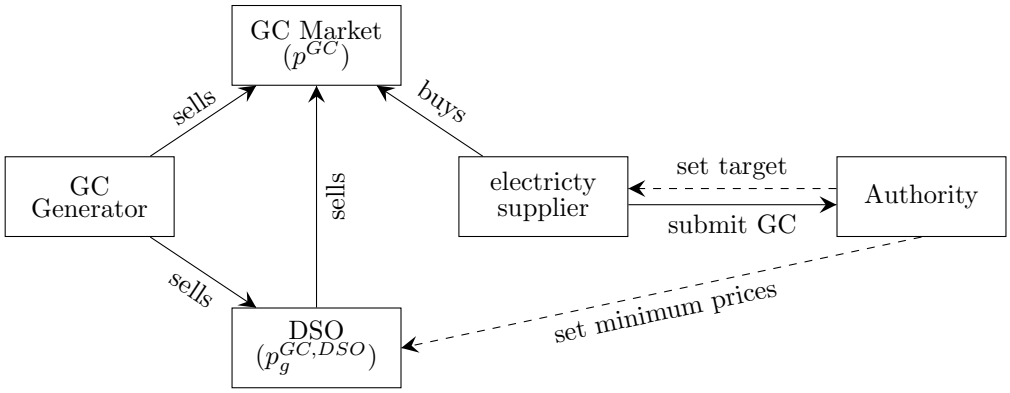

Figure 5: Green certificate trading scheme

In this illustration, we focus on the long-run equilibrium between the different GenCos. The optimization problem faced by a profit-maximizing, price-taking GenCo i is as follows:

$$\max_{cap_{i,g}, gen_{i,g,t}, q_{i,g}^{DSO}, q_{i,g}^{MAR}} \quad \sum_g \Big[\sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + q_{i,g}^{MAR} p^{GC} + q_{i,g}^{DSO} P_g^{GC,DSO} - cap_{i,g} C_g^{CAPEX} - \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big] \tag{10a}$$

$$s.t. \quad AF_{g,t} cap_{i,g} - gen_{i,g,t} \geq 0 \quad \forall g, t \tag{10b}$$

$$\sum_t (R_g gen_{i,g,t} \Delta_t) - q_{i,g}^{DSO} - q_{i,g}^{MAR} \geq 0 \quad \forall g \tag{10c}$$

$$gen_{i,g,t} \geq 0 \quad \forall g, t \tag{10d}$$

$$cap_{i,g}, q_{i,g}^{DSO}, q_{i,g}^{MAR} \geq 0 \quad \forall g. \tag{10e}$$

The problem is very similar to the example presented in Section 2.1. In contrast to the problem described in Section 2.1, we here assume that each agent i can invest in different technologies g. Moreover, we introduce an availability factor $AF_{g,t}$ to account for the renewable generators' weather-dependent ability to generate electricity in different moments in time. Aside from the revenues from selling their electricity in the market for electricity, the generators of renewable energy receive additional revenues by either selling their green certificates to the DSO or directly to the market. Here, $q_{i,g}^{DSO}$ and $q_{i,g}^{MAR}$ represent the number of certificates received by agent i for a technology g that are sold to the DSO and the market for green certificates respectively. Eq. (10c) ensures that each agent cannot sell more certificates than those received by generating renewable electricity.

In addition, decision variables of different agents (i.e., different GenCos) now come together in two linking constraints:

$$\sum_i \sum_g (gen_{i,g,t}\Delta_t) = D_t\Delta_t \quad (p_t^{el}) \quad \forall t, \tag{11}$$

$$\sum_i \sum_g (q_{i,g}^{DSO} + q_{i,g}^{MAR}) \geq FR \sum_t (D_t\Delta_t) \quad (p^{GC}). \tag{12}$$

Eq. (11) again states that total generation should equal the demand for electricity in every time step, whereas Eq. (12) guarantees that sufficient green certificates are generated for the suppliers to meet their obligation. Note that in the presented formulation of the equilibirum problem, the suppliers are not represented as explicit agents. Rather, the suppliers are represented by an inelastic demand for green certificates. In addition, also the DSO is not explicitly represented as a separate agent. This because we assume that the DSO directly sells all the certificates he has acquired to the market (i.e., the amount of certificates sold by the generators to the DSO equals the amount of certificates sold by the DSO to the market).

3.1.2 Numerical Example.

In this section, the equilibrium problem (optimization problems defined by Eq. (10a)-(10e) with linking constraints Eq. (11)-(12)) for a system inspired by the Belgian electricity system is implemented as an MCP and solved using the Path solver. No existing capacity is considered, i.e., we are interested to find the long-run equilibrium for a given RES quota (i.e., fraction FR) and guaranteed minimum prices for green certificates of different origins.

Five technologies are considered: three conventional and dispatchable generators (base, mid and peak-load), and two types of RES (onshore wind and solar PV). The considered techno-economic characteristics of these technologies are presented in Tab. 1. The demand for electricity is assumed inelastic and is based on historical data provided by the Belgian transmission system operator Elia. The average demand for electricity in the considered system is about 10 GW, with peaks up to around 13.6 GW. Similar to the time series for the demand, the time series for the generation by wind and solar PV are based on historical data provided by (55). To restrict computation times, the operations of an entire year are approximated by a representative set of 24 individual hours, which are selected using the methodology described in (56).

Parameter	Unit	Base	Mid	Peak	Wind	PV
C^{CAPEX}	$\frac{EUR}{kW \cdot a}$	160	140	120	170	170
C^{OPEX}	$\frac{EUR}{MWh}$	40	50	75	0	0
$\overline{AF}$	%	100	100	100	25.6	11.5
R	–	-	-	-	0.5	0.7
$P^{GC,DSO}$	$\frac{EUR}{GC}$	-	-	-	50	183

Table 1: Techo-economic characteristics of the considered technologies

The results of the simulations for varying values of the RES quota are presented in Fig. 6. This upper part of this figure shows the market price for green certificates, whereas the bottom part show the installed capacity of solar PV panels and wind turbines.

Three zones of RES quota can be identified in Fig. 6, as indicated by the vertical lines. In a first zone, corresponding to low RES quota, the RES quota constraint (Eq. (12)) is not binding, i.e., the guaranteed minimum price for green certificates provides sufficient incentives to invest in more renewable capacity than needed to reach the RES quota. Given the techno-economic assumptions adopted in this case, the guaranteed support of 183 $\frac{EUR}{GC}$ for solar PV panels results in an equilibrium in which about 12 GW of solar PV capacity is installed. This installed capacity is sufficient to achieve a RES quota of almost 10%. As a result, if the actual RES quota is below this value, there is an excess of green certificates in the market and the market price goes to zero. Thus, in this zone, the equilibrium is not impacted by the quota constraint, and hence, the quota constraint could equally well be removed from the model.

In the second zone, the investments induced solely by the guaranteed minimum price for green certificates are no longer sufficient to achieve the RES quota, i.e., the RES quota constraint becomes binding and the market price for green certificates becomes positive. More specifically, the market price

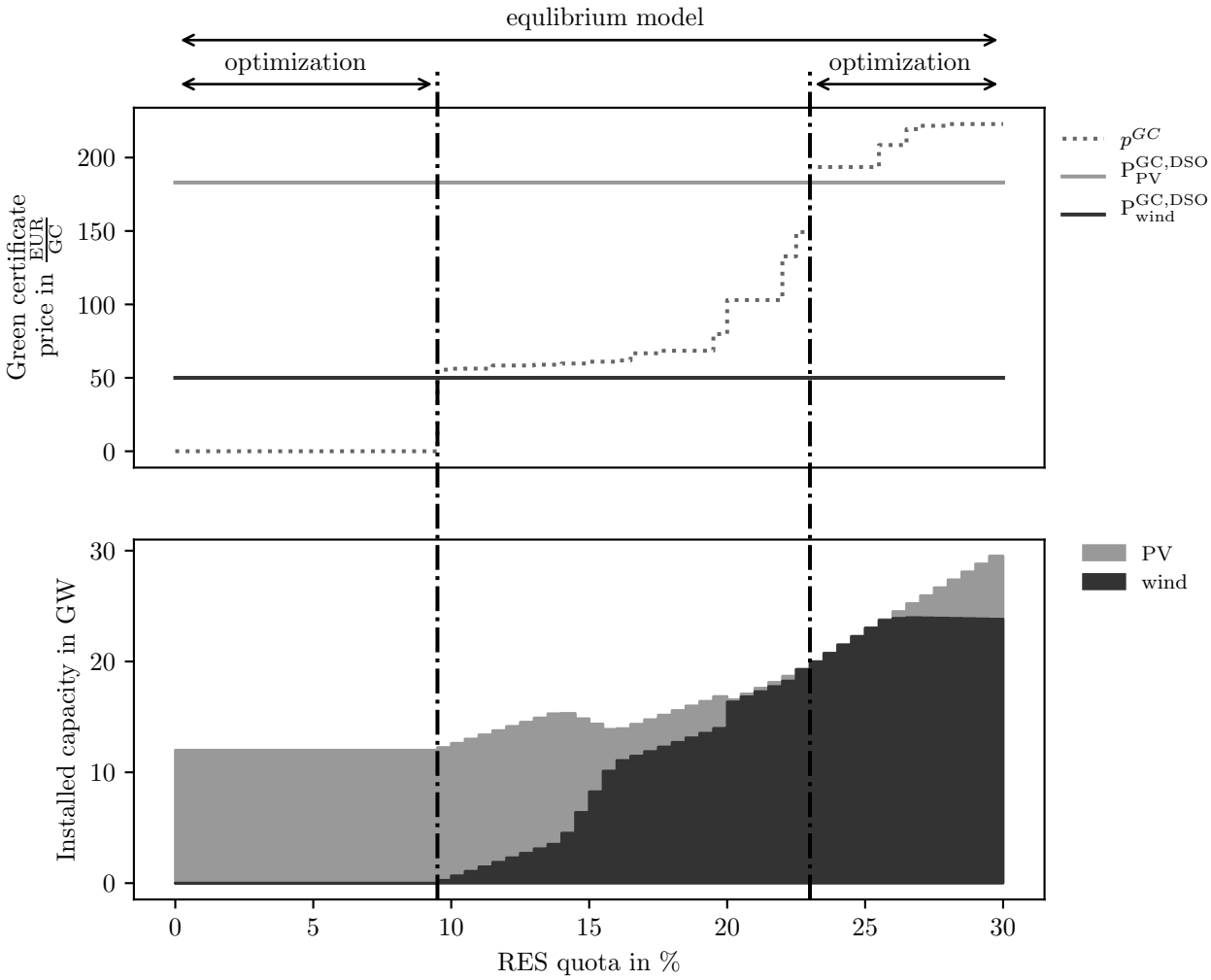

Figure 6: Market price for green certificates (top figure), and renewable capacity mix (bottom figure) for an increasing RES quota

for green certificates jumps to the value that is just sufficient to achieve the RES quota. In terms of investments, it can be observed that in this zone, wind turbines start to become part of the equilibrium solution (and later even start to displace solar PV capacity). As can be observed in Fig. 6, for RES quotas between 10% and 23%, the market price for green certificates lies between the minimum prices offered to wind and solar PV respectively. In this zone, owners of PV installations will remain to sell their green certificates to the DSO at the minimum price of 183 $\frac{EUR}{GC}$, whereas owners of wind turbines will sell their green certificates to suppliers at the market price (which is lower than the minimum price for green certificates offered to certificates originating from solar PV). With increasing amount of zero-marginal cost RES being pushed into the system, the electricity prices during moments in which the RES are generating electricity tend to become lower. Consequently, the revenues per unit of installed RES capacity from selling electricity in the electricity market decrease with an increasing penetration of RES (this is referred to as the so-called "self-cannibalization" effect). As a result, the required support (in the form of the green certificate price) to achieve the RES quota increases with more ambitious RES quota.

In the third zone, corresponding to the highest RES quota, the market price of green certificates reaches a point where it exceeds the minimum prices for green certificates for both solar PV and wind power, and increasing investments in both solar PV and wind can be observed. Within this zone, both owners of wind turbines and solar PV installations decide to sell their green certificates to suppliers at the market price. Hence, within this zone, the minimum support prices no longer impact the equilibrium. Therefore, the minimum support prices could be removed from the model.

3.1.3 Why the Equilibrium Problem Cannot be Cast into an Optimization Problem

We now show that this equilibrium problem (in its general form) cannot be cast into an optimization model and correspondingly solved. According to Condition 1 (Eq. (7)), in order for this problem to be cast into an optimization model, there should exist a function $F(\mathbf{x})$ and functions $a_i(\mathbf{x}_{-i}, \boldsymbol{\gamma})$, such that Eq. (7) holds. Recall that in Eq. (7), the vector $\mathbf{x}$ refers to the primal decision variables of all agents (i.e., *cap*, *gen*, q^{MAR} and q^{DSO}), while the vectors $\mathbf{x}_i$ and $\mathbf{x}_{-i}$ respectively refer to the decision variables of agent i and the decision variables of all agents except agent i. Finally, the vector $\boldsymbol{\gamma}$ refers to the dual variables of the linking constraints (i.e., p^{el} and p^{GC}). Given the objective function of each agents' optimization problem (Eq. (10a)), and the linking constraints (Eq. (11)-(12)), Condition 1 becomes:

$$\begin{aligned}
&\sum_g \Big[\sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + q_{i,g}^{MAR} p^{GC} + q_{i,g}^{DSO} P_g^{GC,DSO} \\
&- cap_{i,g} C_g^{CAPEX} - \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big] \\
=&F(cap, gen, q^{DSO}, q^{MARK}) \\
&+ \sum_t \Big(p_t^{el} \big(\sum_i \sum_g (gen_{i,g,t} \Delta_t) - D_t \Delta_t) \big) \Big) \\
&+ p^{GC} \Big(\sum_i \sum_g \big(q_{i,g}^{DSO} + q_{i,g}^{MAR} \big) - \sum_t \big(D_t \Delta_t FR \big) \Big) \\
&+ a_i(cap_{-i,g}, gen_{-i,g,t}, q_{-i,g}^{DSO}, q_{-i,g}^{MARK}, p_t^{el}, p^{GC})
\end{aligned} \tag{13}$$

By splitting up the terms related to the linking constraints into terms relating to agent i and the terms related to the other agents, one obtains:

$$\begin{aligned}
&\sum_g \Big[\sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + q_{i,g}^{MAR} p^{GC} + q_{i,g}^{DSO} P_g^{GC,DSO} \\
&- cap_{i,g} C_g^{CAPEX} - \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big] \\
=&F(cap, gen, q^{DSO}, q^{MARK}) \\
&+ \sum_g \sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + \sum_t \Big(p_t^{el} \big(\sum_{-i} \sum_g (gen_{i,g,t} \Delta_t) - D_t \Delta_t \big) \Big) \\
&+ \sum_g q_{i,g}^{MAR} p^{GC} + \sum_g q_{i,g}^{DSO} p^{GC} + p^{GC} \Big(\sum_{-i} \sum_g \big(q_{-i}^{DSO} + q_{-i}^{MAR} \big) - \sum_t \big(D_t \Delta_t FR \big) \Big) \\
&+ a_i(cap_{-i,g}, gen_{-i,g,t}, q_{-i,g}^{DSO}, q_{-i,g}^{MARK}, p_t^{el}, p^{GC})
\end{aligned} \tag{14}$$

By recognizing that the function F() is the only remaining place where terms containing the primal decision variables of generation company i can enter the right-hand side of the equation, we propose a function F() as follows:

$$\begin{aligned}
&F(cap, gen, q^{DSO}, q^{MARK}) \\
&= - \sum_g \sum_i cap_i C_g^{CAPEX} - \sum_g \sum_i \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big) + \sum_g \sum_i q_{i,g}^{DSO} B_g
\end{aligned} \tag{15}$$

, and substituting this into Eq. (14), one becomes, after rearranging, the following equation:

$$\begin{aligned}
&\sum_g \Big[\sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + q_{i,g}^{MAR} p^{GC} + q_{i,g}^{DSO} P_g^{GC,DSO} \\
&- cap_{i,g} C_g^{CAPEX} - \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big] \\
=&\sum_g \Big[\sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + q_{i,g}^{MAR} p^{GC} + q_{i,g}^{DSO} (p^{GC} + B_g) - cap_{i,g} C_g^{CAPEX} - \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big] \\
&- \sum_{-i} \sum_g cap_{-i,g} C_g^{CAPEX} - \sum_{-i} \sum_g \sum_t (gen_{-i,g,t} C_g^{OPEX} \Delta_t) + \sum_{-i} \sum_g (q_{-i,g}^{DSO} B_g) \\
&+ \sum_t \Big(p_t^{el} \big(\sum_{-i} \sum_g (gen_{i,g,t} \Delta_t) - D_t \Delta_t \big) \Big) \\
&+ p^{GC} \Big(\sum_{-i} \sum_g \big(q_{-i,g}^{DSO} + q_{-i,g}^{MAR} \big) - \sum_t \big(D_t \Delta_t FR \big) \Big) \\
&+ a_i(cap_{-i,g}, gen_{-i,g,t}, q_{-i,g}^{DSO}, q_{-i,g}^{MARK}, p_t^{el}, p^{GC})
\end{aligned} \tag{16}$$

Note that only the first line of the right-hand side of Eq. (16) now contains decision variables related to agent i. Hence, by selecting the function $a_i()$ as:

$$\begin{aligned}
&a_i(cap_{-i,g}, gen_{-i,g,t}, q_{-i,g}^{DSO}, q_{-i,g}^{MARK}, p_t^{el}, p^{GC}) \\
=&\sum_{-i} \sum_g cap_{-i,g} C_g^{CAPEX} + \sum_{-i} \sum_g \sum_t (gen_{-i,g,t} C_g^{OPEX} \Delta_t) - \sum_{-i} \sum_g (q_{-i,g}^{DSO} B_g) \\
&- \sum_t \Big(p_t^{el} \big(\sum_{-i} \sum_g (gen_{-i,g,t} \Delta_t) - D_t \Delta_t \big) \Big) \\
&- p^{GC} \Big(\sum_{-i,g} \big(q_{-i,g}^{DSO} + q_{-i,g}^{MAR} \big) - \sum_t \big(D_t \Delta_t FR \big) \Big)
\end{aligned} \tag{17}$$

, and substituting this in Eq. (16), one becomes:

$$\begin{aligned}
&\sum_g \Big[\sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + q_{i,g}^{MAR} p^{GC} + q_{i,g}^{DSO} P_g^{GC,DSO} \\
&- cap_{i,g} C_g^{CAPEX} - \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big] \\
=&\sum_g \Big[\sum_t (gen_{i,g,t} p_t^{el} \Delta_t) + q_{i,g}^{MAR} p^{GC} + q_{i,g}^{DSO} (p^{GC} + B_g) \\
&- cap_{i,g} C_g^{CAPEX} - \sum_t (gen_{i,g,t} C_g^{OPEX} \Delta_t) \Big]
\end{aligned} \tag{18}$$

From this equation, it can be seen that only if it would be possible to determine a value B_g, such that $p^{GC} + B_g = P_g^{GC,DSO}$ holds, the optimization problem, with objective function F(), would solve the equilibrium problem[13]. However, the market price for green certificates (p^{GC}) is not known a priori, making it generally not possible to determine the value for parameter B_g such that the optimization model solves the equilibrium problem.

From a less mathematical perspective, one could also observe that the equilibrium problem cannot be cast into an equivalent optimization model because the equilibrium problem is not in accordance with the first assumption inherently made in system optimization models, as stipulated in Section 2.2.1. More specifically, not all variables appearing in the linking constraint ensuring the achievement of the RES quota (Eq. (12)) participate in a paid-as-cleared market for green certificates. Although the variables $q_{i,g}^{DSO}$ and $q_{i,g}^{MAR}$ both contribute equally to meeting the linking constraint for the supply of green certificates, the variable $q_{i,g}^{DSO}$ does not receive the market price.

Aside from formulating the equilibrium problem as an MCP and solving the MCP, another way to solve the equilibrium problem is to make use of an iterative algorithm relying on the fact that the optimization problem defined above is almost equivalent to the equilibrium problem. In such an iterative

algorithm, an initial value for B_g is estimated, the optimization problem is solved, and B_g is updated based on the observed market prices for green certificates in that simulation. Subsequently, the optimization models is solved again, the values for B_g are again updated, and this process is continued until convergence is reached.

In is of interest to note that there are situations in which an optimization model could directly provide the solution to the equilibrium problem. A first situation is if one knows a priori that the minimum prices offered by the DSO will incentivize more investments in renewable generators than needed to reach the imposed target, i.e., when one is in the first zone described in Section 3.1.2. In this case, the market price for green certificates will be zero; and hence the parameter value B_g can be taken equal to the price offered by the DSO (and the RES quota constraint could be dropped from the model formulation).

A second situation is when one knows a priori that the market price for green certificates will be higher than the minimum price offered for green certificates for all technologies, i.e., when one is in the third zone described in Section 3.1.2. In this case, one can safely assume that all green certificates will be sold directly to the market. Hence, the term $+q_{i,g}^{DSO}(p^{GC} + B_g)$ in the objective function of the optimization problem as well as the term $+q_{i,g}^{DSO} P_g^{GC,DSO}$ in the objective function of agent i can be removed, making both problems equivalent.

However, in many cases, it will not be straightforward to determine a priori whether one of these situations will occur. In addition, one still would need to use an iterative algorithm to find the solutions in the middle zone described in Section 3.1.2. It must furthermore be noted that the presented case is simplified. In actual large-scale cases, other factors might complicate things further. For instance, if we would aim to find the intertemporal equilibrium instead of the long-run equilibrium (corresponding to the greenfield situation), it could perfectly well be that generators of renewable electricity sell their electricity at the guaranteed minimum price for a number of years, after which the market price for green certificates exceeds the minimum price, and generators switch to sell their electricity to the market. In these situations, solving the equilibrium problem using an iterative algorithm, in which an optimization problem is solved repeatedly, is likely to become highly cumbersome. In contrast, if the equilibrium problem is formulated as an MCP, it could still be directly solved.

3.2 Other Illustrations

3.2.1 Net Metering.

Consider an equilibrium problem where, in addition to the generators competing on the wholesale level, residential consumers j can decide to invest in solar PV panels. For sake of simplicity, we assume that all consumers have the option to invest in solar PV panels. We furthermore assume that all residential consumers have net metering contracts with their suppliers. The suppliers are not explicitly modeled, but we assume that these suppliers offer a single retail price $p_t^{el,RT}$ to all consumers/prosumers regardless of their consumption (and generation) patterns, i.e., consumers are billed based on their annual net electrical energy consumption.

Each consumer j then faces the problem of minimizing its costs for electricity by deciding whether to buy all electricity via the suppliers or generate some electricity themselves by investing in solar PV panels:

$$\min_{cap_j^{PV}, gen_{j,t}^{PV}} \quad p^{el,RT} \sum_t \left((D_{j,t} - gen_{j,t}^{PV}) \Delta_t \right) + C_j^{INV,PV} cap_j^{PV}, \tag{19a}$$

$$s.t. \quad gen_{j,t}^{PV} \leq cap_j^{PV} CF_{j,t}^{PV} \quad \forall t, \tag{19b}$$

$$cap_j^{PV}, gen_{j,t}^{PV} \geq 0. \tag{19c}$$

Here, $gen_{j,t}^{PV}$ and $D_{j,t}$ are respectively the average electrical power generated and consumed by consumer j during time interval t. In addition, the parameter $CF_{j,t}^{PV}$ represents the capacity factor of the solar PV panels within this time interval.

The optimization problem faced by each GenCo i operating in the wholesale markets is as follows:

$$\max_{cap_i, gen_{i,t}} \quad \sum_t \left(gen_{i,t} (p_t^{el,WS} - C_i^{OPEX}) \Delta_t \right) - cap_i C_i^{CAPEX} \tag{20a}$$

$$s.t. \quad gen_{i,t} \leq cap_i \quad \forall t, \tag{20b}$$

$$cap_i, gen_{i,t} \geq 0, \tag{20c}$$

, where $p_t^{el,WS}$ represents the average wholesale electricity price during time interval t.

The linking constraint representing the balance between supply and demand is as follows:

$$\sum_i gen_{i,t}\Delta_t = \sum_j \left((D_{j,t} - gen_{j,t}^{PV})\Delta_t\right) \quad (p_t^{el,WS}) \quad \forall t. \tag{21}$$

For sake of simplicity, we have assumed here that the total demand for electricity comes from residential consumers.

Finally, we assume that the retail price is determined by the suppliers as the volume weighted wholesale price of the net consumption of all consumers increased by a margin for suppliers (T^{supp}) and the transmission and distribution tariffs (T^{trans} and T^{distr}).

$$p^{el,RT} = \frac{\sum_{j,t}\left((D_{j,t} - gen_{j,t})p_t^{el,WS}\Delta_t\right)}{\sum_{j,t}\left((D_{j,t} - gen_{j,t})\Delta_t\right)} + T^{supp} + T^{trans} + T^{distr} \tag{22}$$

Defining an optimization problem that directly solves this equilibrium problem is not possible. This because the first assumption inherently made by optimization models is again violated in this example. Specifically, in this equilibrium problem, the solar PV generation is not remunerated at the paid-as-cleared wholesale market price (i.e., the dual variable of Eq. (21)), even though the variable for solar generation does contribute to this linking constraint. As such, an optimization model cannot directly simulate the market distortion related to having average prices. Note that incorporating the margin for suppliers as well as the transmission and the distribution charges poses no problem as long as these additional charges are assumed independent of the market outcome (i.e., the market prices).

3.2.2 Decision Making under Uncertainty.

In liberalized and deregulated electricity markets, GenCos face many uncertainties. These include among others the uncertainty regarding future fuel prices, technological development, demand growth, policy interventions, and the decisions made by competitors (46, 57). GenCos can account for these uncertainties in the investment planning problem by considering a number of possible scenarios, where each scenario represents one possible realization of the uncertain parameters.

Assuming risk-neutral GenCos, the objective of each GenCo i is to maximize its expected profits. Given a set of scenarios $w \in \Omega$, each with a probability of π_w, the objective function becomes:

$$\max_{cap_i, gen_{i,t,w}} \quad \sum_w \pi_w \Big[\sum_t \left(gen_{i,t,w}(p_{t,w}^{el} - C_{i,w}^{OPEX})\Delta_t\right) - cap_i C_i^{CAPEX} \Big]. \tag{23}$$

The market clearing constraint is now imposed for every time step and scenario.

$$\sum_i gen_{i,t,w}\Delta_t = D_{t,w}\Delta_t \quad (\pi_w p_{t,w}^{el}) \quad \forall t, w \tag{24}$$

This stochastic equilibrium problem can be cast into a stochastic system optimization problem in which the objective function is to maximize the expected total surplus (as shown in (38)). Note that in such a stochastic surplus maximization problem, the dual variable of the linking constraint (Eq. (24)) can be interpreted as the probability-weighted electricity price, as indicated between brackets. Following the first necessary condition for an equilibrium problem to be cast in an optimization problem (Eq. (7)), the objective function of the GenCo should thus contain the terms $\sum_{t,w} gen_{i,t,w}\Delta_t \pi_w p_{t,w}^{el}$. As can be seen from Eq. (23), this is indeed the case in this example.

However, a first issue arises if the expectations of the agents towards the uncertain parameters are not homogeneous, i.e., when different agents attach a different probability to a particular scenario (e.g., one agent believes high carbon prices in the future are unlikely while another agent does not). In this case, an optimization model cannot be used to compute the equilibrium. In this equilibrium problem, the agents do participate in a paid-as-cleared electricity, which is in correspondence to the linking constraint demanding a balance between demand and supply in every time step and scenario (Eq. (24)). However, the different agents will value the hypothetical revenues that would be obtained in a given scenario differently as they attach different probabilities to the possibility that that scenario will effectively be realized. This is not in line with the third inherent assumption made in optimization models (see Section 2.2.1).

A second issue arises if agents are assumed risk averse. Consider as an example that each GenCo aims to maximize their expected profits subject to the constraint that the potential loss cannot exceed a certain threshold T_i. The objective function of each agent remains to be represented by Eq. (23), but now each agent faces additional constraints:

$$\sum_t \left(gen_{i,t,w}(p^{el}_{t,w} - C^{OPEX}_{i,w})\Delta_t\right) - cap_i C^{CAPEX}_i \geq -T_i \quad \forall w. \tag{25}$$

These constraints contain dual variables. Therefore, these constraints cannot be represented directly in a system optimization model, as indicated by Condition 2 in Section 2.2.2. A similar result can be found when more advanced risk measures such as the conditional value at risk are used to model risk-averse behavior. For a detailed treatment of risk in equilibrium investment planning problems, we refer to (46, 34, 58).

Most energy-system and power-system optimization models do not endogenously evaluate the uncertainty and the associated risk. Rather, an expectation of the involved risk is frequently reflected in the choice of the discount rate used. This discount rate then reflects both the cost of acquiring capital and a risk premium. Because both the cost of acquiring capital and the involved risk can differ for different agents and different technologies/projects, the discount rates (i.e., the hurdle rates) used to evaluate the profitability of possible investments differs from project to project.

Given that different discount rates d_i are used for different projects, the objective function of the multi-period investment planning problem faced by GenCo i becomes:

$$\max_{cap_{i,y}, gen_{i,y,t}} \quad \sum_y \frac{1}{(1+d_i)^y}\left(\sum_t \left(gen_{i,y,t}(p^{el,WS}_{y,t} - C^{OPEX}_{i,y})\Delta_t\right) - cap_i C^{CAPEX}_{i,y}\right) \tag{26}$$

Here, the index y is added to represent different years in the planning horizon. For simplicity of notation, we again assume that each agent can invest only in a single technology, and that the discount rate is only dependent on the characteristics of the agent and the choice of technology.

Due to the fact that different agents apply different discount rates d_i, the perceived value of generating a certain amount of electricity in a specific future time period is different for each agent despite the fact that there is a unique price for electricity in each period. This violates the third inherent assumption made in optimization models (in a similar fashion as the stochastic model with heterogeneous expectations detailed above). As a result, optimization models cannot be used to compute the equilibrium when different projects are evaluated using different discount rates[14]. For a detailed discussion of this issue, we refer to (35).

3.2.3 Additional Illustrations.

While it is not our ambition to provide an exhaustive overview of market designs, policy interventions and behavioral traits that require MCP or other types of models, Tab. 2 lists some equilibrium problems treated in this text or encountered in the literature that cannot be cast into and solved using an optimization model. In addition, this table indicates for each of these problems which of the inherent assumptions in system optimization models is violated.

Implicit assumption violated	Market design	Policy intervention	Agent behavior
1. All variables contributing to a certain linking constraint participate in a paid-as-cleared-market	Net metering, Average price contracts (47)	Minimum price for green certificates, Feed-in tariffs (43), VAT tax	-
2. The endogenously determined market price cannot determine the value of variables not appearing in the corresponding linking constraint	Net metering, Average price contracts (47)	Grandfathering of emission allowances for new installations (45, 46)	-
3. All agents have the same valuation of the revenues from participating in the market represented by a linking constraint	-	-	Heterogeneous perception of uncertainties, Risk-averse investors (34, 46, 58)
4. The decision space of each agent is not dependent on endogenously determined market prices	-	-	Risk-averse investors (34, 46, 58), maximum payback time

Table 2: Examples of market designs, policy interventions and agent behavior that cannot be represented in optimization models. The first column indicates which inherent assumption of the optimization model is violated for each of the presented examples. See Section 2.2.1 for a more detailed discussion regarding these inherent assumptions.

4 Summary and Conclusions

In the context of liberalized and deregulated energy markets, long-term energy-system or power-system optimization models are used for two distinct purposes. A first is to address normative questions by analyzing how the optimal transition of the energy/electricity system looks like under certain assumptions. A second is to describe the likely/expected evolution of the energy/electricity system when certain policies are put into place (i.e., a descriptive perspective is taken). In this regard, optimization models rely on the fact that in competitive markets the surplus is maximized in the market equilibrium. As such, the market equilibrium can be computed by maximizing total surplus.

While optimization models can be used to compute the market equilibrium in markets in which not all conditions for perfect competition are satisfied, there are a number of limitations for the use of optimization models. One well-known limitation is that optimization models implicitly assume price-taking agents. However, even if all agents are assumed price takers, optimization models face restrictions in terms of representing certain market designs, policy interventions or behavioral characteristics of agents, hence limiting their applicability for generating descriptive scenarios.

This paper aims to help modelers to gain insights into the ability and limitations of optimization models for computing market equilibria. To this end, we have provided a framework (both mathematical and non-mathematical) that allows modelers to check whether a certain equilibrium problem can be cast into an optimization model. In addition, we have presented a number of topical examples of equilibrium problems that cannot be cast into a system optimization model. In this paper, we have restricted ourselves to equilibrium problems with price-taking agents. For an overview of different techniques for solving equilibrium problems where agents behave strategically, we refer to (37). Insights into the abilities and limitations of optimization models are important to decide on a long-term strategy for the type of model to develop. This is particularly important given that setting up energy-system and power-system optimization models is resource intensive.

According to Condition 1 (Eq. (7)), in order for the equilibrium problem (Eq. (1)-(3)) to be cast into an optimization model, there should exist a function $F(\mathbf{x})$ and functions $a_i(\mathbf{x}_{-i}, \boldsymbol{\gamma})$, such that Eq. (7) holds. Recall that in Eq. (7), the vector $\mathbf{x}$ refers to the primal decision variables of all agents (i.e., *cap*, *gen* and q), while the vectors $\mathbf{x}_i$ and $\mathbf{x}_{-i}$ respectively refer to the decision variables of agent i and the decision variables of all agents except agent i. Finally, the vector $\boldsymbol{\gamma}$ refers to the dual variables of the linking constraints (i.e., p^{el}).

Given the linking constraints of the equilibrium problem (Eq. (3)), applying Condition 1 (Eq. (7)) for generation company agent i, provides following condition:

$$\begin{aligned}
&\sum_t (gen_{i,t}\Delta_t p_t^{el}) - cap_i C_i^{CAPEX} - \sum_t (gen_{i,t} C_i^{OPEX} \Delta_t) \\
=&F(cap, gen, q) \\
&+ \sum_t \Big(p_t^{el} \big(\sum_i (gen_{i,t}\Delta_t) - q_t \Delta_t \big) \Big) \\
&+ a_i(cap_{-i}, gen_{-i,t}, q_t, p_t^{el})
\end{aligned} \tag{27}$$

By splitting up the terms related to the linking constraints into terms relating to generation company i and the terms related to the other agents, one obtains:

$$\begin{aligned}
&\sum_t (gen_{i,t}\Delta_t p_t^{el}) - cap_i C_i^{CAPEX} - \sum_t (gen_{i,t} C_i^{OPEX} \Delta_t) \\
=&F(cap, gen, q) \\
&+ \sum_t (gen_{i,t}\Delta_t p_t^{el}) + \sum_t \Big(p_t^{el} \big(\sum_{-i} (gen_{-i,t}\Delta_t) - q_t \Delta_t \big) \Big) \\
&+ a_i(cap_{-i}, gen_{-i,t}, q_t, p_t^{el})
\end{aligned} \tag{28}$$

By recognizing that the function F() is the only remaining place where terms containing the primal decision variables of generation company i can enter the right-hand side of the equation, we propose a function F() as follows:

$$F(cap, gen, q) = -\sum_i cap_i C_i^{CAPEX} - \sum_i \sum_t \big(gen_{i,t} C_i^{OPEX} \Delta_t \big) + G(q) \tag{29}$$

, where G() is a function of the decision variable of the consumer. By subsituting the proposed function F() into Eq. (28), one becomes, after rearranging, the following equation:

$$\begin{aligned}
&\sum_t (gen_{i,t}\Delta_t p_t^{el}) - cap_i C_i^{CAPEX} - \sum_t (gen_{i,t} C_i^{OPEX}\Delta_t) \\
=&\sum_t (gen_{i,t}\Delta_t p_t^{el}) - \sum_i cap_i C_i^{CAPEX} - \sum_t \big(gen_{i,t} C_i^{OPEX}\Delta_t\big) \\
&+ G(q) + \sum_t \Big(p_t^{el}\big(\sum_{-i}(gen_{-i,t}\Delta_t) - q_t\Delta_t\big)\Big) - \sum_{-i}\sum_t \big(gen_{-i,t} C_{-i}^{OPEX}\big) \\
&+ a_i(cap_{-i}, gen_{-i,t}, q_t, p_t^{el})
\end{aligned} \tag{30}$$

By selecting the function $a_i()$ as:

$$a_i(cap_{-i}, gen_{-i,t}, q, p_t^{el}) - G(q) - \sum_t \Big(p_t^{el}\big(\sum_{-i}(gen_{-i,t}\Delta_t) - q_t\Delta_t\big)\Big) + \sum_{-i}\sum_t \big(gen_{-i,t} C_{-i}^{OPEX}\big) \tag{31}$$

, and substituting this in Eq. (30), it can be seen that the condition holds for generation company i (and by extension for all generation companies).

However, Condition 1 (Eq. (7)) should also hold for the consumer. Given the consumer's objective function (Eq. (2a)), the linking constraint (Eq. (3)), and the proposed function F() (Eq. (29)), Condition 1 becomes:

$$\begin{aligned}
&\sum_t \Big(\int_0^{q_t} f_{d,t}^{-1}(q_t')dq_t'\Delta_t\Big) - \sum_t \big(q_t\Delta_t p_t^{el}\big) \\
=&+ G(q) - \sum_i cap_i C_i^{CAPEX} - \sum_i\sum_t \big(gen_{i,t} C_i^{OPEX}\Delta_t\big) \\
&+ \sum_t \Big(p_t^{el}\big(\sum_i(gen_{i,t}\Delta_t) - q_t\Delta_t)\big)\Big) \\
&+ b(cap_i, gen_{i,t}, p_t^{el})
\end{aligned} \tag{32}$$

After rearranging, this gives:

$$\begin{aligned}
&\sum_t \Big(\int_0^{q_t} f_{d,t}^{-1}(q_t')dq_t'\Delta_t\Big) - \sum_t \big(q_t\Delta_t p_t^{el}\big) \\
=&+ G(q) - \sum_t \big(q_t\Delta_t p_t^{el}\big) \\
&- \sum_i cap_i C_i^{CAPEX} - \sum_i\sum_t \big(gen_{i,t} C_i^{OPEX}\Delta_t\big) + \sum_t \Big(p_t^{el}\big(\sum_i(gen_{i,t}\Delta_t)\big)\Big) \\
&+ b(cap_i, gen_{i,t}, p_t^{el})
\end{aligned} \tag{33}$$

By taking the function G() as:

$$G(q) = \sum_t \Big(\int_0^{q_t} f_{d,t}^{-1}(q_t')dq_t'\Delta_t\Big) \tag{34}$$

, and the function b() as:

$$b(cap_i, gen_{i,t}, p_t^{el}) = \sum_i cap_i C_i^{CAPEX} + \sum_i\sum_t \big(gen_{i,t} C_i^{OPEX}\Delta_t\big) - \sum_t \Big(p_t^{el}\big(\sum_i(gen_{i,t}\Delta_t)\big)\Big) \tag{35}$$

, Condition 1 is shown to be satisfied also for the consumer.

Hence, the solution of a system optimization problem with the objective functionto maximize equal to:

$$F(cap, gen, q) = \sum_t \Big(\int_0^{q_t} f_{d,t}^{-1}(q_t')dq_t'\Delta_t\Big) - \sum_i cap_i C_i^{CAPEX} - \sum_i\sum_t \big(gen_{i,t} C_i^{OPEX}\Delta_t\big) \tag{36}$$

, and as constraints the constraints faced by each agent as well as the linking constraint, is a solution to the equilibrium problem.

Endnotes

1. Specifically, this occurs if there are multiple inverse demand/supply functions that are asymmetric, i.e., the partial derivatives of the inverse demand/supply functions of two commodities to a change in the quantity of the other commodity are not equal. Asymmetric inverse demand/supply functions only occur if there are multiple inverse/demand functions and some of these functions have cross-price elasticities. Multiple inverse demand functions with cross-price elasticities may be needed if there are multiple commodities (e.g., a high price for electricity might increase the demand for natural gas) or different time steps (e.g., a high price for electricity in a certain time period might reduce the demand for electricity in this time period but also increase the demand for electricity in subsequent time periods). Typically, energy-system and power-system optimization models consider the demand to be fixed or only consider own-price elasticities. We do not focus on this limitation of optimization models in the remainder of this text. For the interested reader, we refer to Chapter 4 of (44) for a more detailed treatment of this limitation of optimization models.

2. As will be discussed later, it might be possible to develop algorithms in which a system optimization model needs to be solved repeatedly until it converges to the equilibrium. However, the optimization model in itself, i.e., without the iterative loop, cannot solve the equilibrium problem. In addition, one might argue that the complementarity constraints of any MCP can be converted to a non-linear problem or a mixed integer program. However, the formulation of those resulting optimization problems reflect the KKT conditions of the MCP and hence differ strongly from the conventionally used system optimization models.

3. Not every equilibrium problem can be formulated as an MCP. Certain type of equilibria, such as Stackelberg equilibria, cannot be formulated as an MCP. Since MCPs generalize the group of linear or convex non-linear optimization problems with continuous variables, these equilibria can also not be solved directly using a non-linear optimization problem with continuous variables. Stackelberg equilibria are commonly formulated as MPECs or EPECs and are often used to model the strategic behavior of an agent that anticipates the reaction of other agents when determining his own actions. These types of equilibria are out of the scope of this work. For a detailed description of these type of problems, we refer to (37, 44).

4. Note that we consider both producers and consumers to be price takers. The price-taking behavior follows from the fact that the price p_t^{el} enters as a parameter in their respective optimization problems, i.e., although the price depends on the agents' decisions and hence is an endogenous variable of the equilibrium problem, within each agent's optimization problem, the price is considered to be a parameter which is independent from its own decisions.

5. For certain type of problems, such as (mixed) integer programs, the KKT conditions are not meaningful, i.e., the KKT conditions are not necessary conditions for the optimal solution. The inability to represent integer variables is a main limitation of MCPs (44).

6. In this example, we do not consider the need for ancillary services such as spinning reserve requirements, and the corresponding interest to operate plants below the rated capacity in order to be able to provide these services.

7. Although it might not be possible to formulate an optimization problem such that the KKT conditions of this optimization problem are identical to the MCP formulation of the equilibrium, it can be possible to determine the equilibrium via iterative algorithms in which the optimization model is solved repeatedly and parameters are adapted. Such iterative algorithms have been used frequently (36) (see e.g., (43, 35) for recent examples). However, the need to solve the optimization problem repeatedly leads to high computational costs. In addition, these iterative algorithms might face convergence issues (35). A detailed discussion of such iterative algorithms and other solution techniques for equilibrium problems are out of the scope of this chapter.

8. If this is not the case, the solution to the optimization problem might violate the constraints faced by one or more agents. In this case, the solution of the optimization problem cannot be a solution to the equilibrium problem.

9. Note that this does not imply that all variables contributing equally to a certain linking constraint should get the same remuneration/cost in total. This because these variables can get additional value (either by appearing in other linking constraints or via exogenously specified costs/revenue terms), which might not be the same for different variables. E.g., certain technologies can get a fixed subsidy on top of their revenues from selling their electricity in the market.

10. As discussed in Section 2.1 presenting the generation expansion planning problem, the optimal

decisions of an agent are dependent on the outcome of the markets (e.g., a generator will not decide to generate electricity unless the price of electricity covers at least its generation costs). However, the decision space of each agent, i.e., the feasible area of its optimization problem, in the presented example is independent of the market prices.
11. Note that this condition must hold for the problem formulation in general, and thus not only at optimality.
12. From a different perspective, the first limitation can be considered to also directly follow from this fact, since if the objective function of the surplus maximization problem could contain dual variables, this would allow adapting the terms containing endogenously determined prices in the KKT conditions by adding additional terms to the objective function.
13. Note that the objective function relates to a maximization problem and can be understood as maximizing negative costs. The parameter B_g can be interpreted as a premium for green certificates, i.e., an additional remuneration on top of the market price.
14. Nevertheless most optimization models approximate the impact of varying discount rates by altering the capital costs of different technologies based on the assumed hurdle rates for the different technologies. As shown in (35), the accuracy of this approximation depends on how the projected revenues vary over the lifetime of the project.

References

[1] European Commission, Overview of competition in energy markets (2012).
URL http://ec.europa.eu/competition/sectors/energy/overview_en.html

[2] D. S. Kirschen, G. Strbac, Fundamentals of power system economics, John Wiley & Sons, Ltd., 2004 (2004).

[3] S. Stoft, Power System Economics: Designing Markets for Electricity, IEEE Press and WILEY-INTERSCIENCE, 2002 (2002).

[4] I. Pérez-Arriaga, P. Linares, Markets vs. regulation: A role for indicative energy planning, available at https://www.iit.comillas.edu/~pedrol/documents/ej08.pdf (2008).

[5] T. Mai, J. Logan, N. Blair, P. Sullivan, M. Bazilian, RE-ASSUME: A Decision Maker's Guide to Evaluating Energy Scenarios, Modeling, and Assumptions, Tech. rep., IEA-RETD (2013).
URL http://iea-retd.org/wp-content/uploads/2013/07/RE-ASSUME_IEA-RETD_2013.pdf

[6] W. McDowall, M. Eames, Forecasts, scenarios, visions, backcasts and roadmaps to the hydrogen economy: A review of the hydrogen futures literature, Energy Policy 34 (11) (2006) 1236 – 1250, hydrogen (2006). doi:https://doi.org/10.1016/j.enpol.2005.12.006.
URL http://www.sciencedirect.com/science/article/pii/S0301421505003460

[7] Integrating Module of the National Energy Modeling System: Model Documentation 2013, Tech. rep., Energy Information Administration (EIA) (2013).
URL https://www.eia.gov/outlooks/aeo/nems/documentation/integrating/pdf/m057(2013).pdf

[8] D. Devogelaer, J. Duerinck, D. Gusbin, Y. Marenne, W. Nijs, M. Orsini, M. Pairon, Towards 100% renewable energy in Belgium by 2050, Tech. rep., VITO, ICEDD and the Belgian Federal Planning Bureau (2013).

[9] K. Karlsson, P. Meibom, Optimal investment paths for future renewable based energy systems—using the optimisation model balmorel, International Journal of Hydrogen Energy 33 (7) (2008) 1777 – 1787 (2008). doi:http://dx.doi.org/10.1016/j.ijhydene.2008.01.031.
URL http://www.sciencedirect.com/science/article/pii/S0360319908000888

[10] A. Chiodi, M. Gargiulo, F. Rogan, J. Deane, D. Lavigne, U. K. Rout, B. P. Ó. Gallachóir, Modelling the impacts of challenging 2050 european climate mitigation targets on ireland's energy system, Energy Policy 53 (0) (2013) 169 – 189 (2013). doi:http://dx.doi.org/10.1016/j.enpol.2012.10.045.
URL http://www.sciencedirect.com/science/article/pii/S0301421512009263

[11] N. Strachan, N. Balta-Ozkan, D. Joffe, K. McGeevor, N. Hughes, Soft-linking energy systems and GIS models to investigate spatial hydrogen infrastructure development in a low-carbon UK energy system, International Journal of Hydrogen Energy 34 (2) (2009) 642 – 657 (2009). doi:http://dx.doi.org/10.1016/j.ijhydene.2008.10.083.
URL http://www.sciencedirect.com/science/article/pii/S036031990801402X

[12] D. Martinsen, J. Linssen, P. Markewitz, S. Vögele, CCS: A future CO2 mitigation option for Germany? A bottom-up approach, Energy Policy 35 (4) (2007) 2110 – 2120 (2007). doi:https://doi.org/10.1016/j.enpol.2006.06.017.

[13] L. Hirth, The optimal share of variable renewables. how the variability of wind and solar power affects their welfare-optimal deployment, Tech. rep., Fondazione Eni Enrico Mattei Working Papers (October 2013).

[14] D. Möst, M. Genoese, A. Esser, O. Rentz, European electricity and emission market modeling ; the design of emission allocation plans and its effects on investment planning, in: 5th International Conference on the European Energy Market (EEM), 2008, pp. 1–6 (May 2008). doi:10.1109/EEM.2008.4579087.

[15] S. Simoes, W. Nijs, P. Ruiz, A. Sgobbi, C. Thiel, Comparing policy routes for low-carbon power technology deployment in eu – an energy system analysis, Energy Policy 101 (Supplement C) (2017) 353 – 365 (2017). doi:https://doi.org/10.1016/j.enpol.2016.10.006.
URL http://www.sciencedirect.com/science/article/pii/S0301421516305493

[16] P. L. Joskow, Capacity payments in imperfect electricity markets: Need and design, Utilities Policy 16 (3) (2008) 159 – 170, capacity Mechanisms in Imperfect Electricity Markets (2008). doi:https://doi.org/10.1016/j.jup.2007.10.003.
URL http://www.sciencedirect.com/science/article/pii/S0957178707000926

[17] L. de Vries, P. Heijnen, The impact of electricity market design upon investment under uncertainty: The effectiveness of capacity mechanisms, Utilities Policy 16 (3) (2008) 215 – 227, capacity Mechanisms in Imperfect Electricity Markets (2008). doi:http://dx.doi.org/10.1016/j.jup.2007.12.002.
URL http://www.sciencedirect.com/science/article/pii/S0957178708000027

[18] H. Höschle, H. L. Cadre, R. Belmans, Inefficiencies caused by non-harmonized capacity mechanisms in an interconnected electricity market, Sustainable Energy, Grids and Networks 13 (2018) 29 – 41 (2018). doi:https://doi.org/10.1016/j.segan.2017.11.002.
URL http://www.sciencedirect.com/science/article/pii/S2352467717301741

[19] H. Höschle, H. L. Cadre, Y. Smeers, A. Papavasiliou, R. Belmans, An admm-based method for computing risk-averse equilibrium in capacity markets, IEEE Transactions on Power Systems 33 (5) (2018) 4819–4830 (Sep. 2018). doi:10.1109/TPWRS.2018.2807738.

[20] A. van Stiphout, K. D. Vos, G. Deconinck, The impact of operating reserves on investment planning of renewable power systems, IEEE Transactions on Power Systems 32 (1) (2017) 378–388 (Jan 2017). doi:10.1109/TPWRS.2016.2565058.

[21] J. Ma, V. Silva, R. Belhomme, D. S. Kirschen, L. F. Ochoa, Evaluating and planning flexibility in sustainable power systems, IEEE Transactions on Sustainable Energy 4 (1) (2013) 200–209 (Jan 2013). doi:10.1109/TSTE.2012.2212471.

[22] E3MLab, Primes model version 6 2016-2017 - detailed model description, Tech. rep. (2016).
URL http://www.e3mlab.eu/e3mlab/PRIMES%20Manual/The%20PRIMES%20MODEL%202016-7.pdf

[23] European Commission, Modelling tools for EU analysis (2019).
URL https://ec.europa.eu/clima/policies/strategies/analysis/models_en

[24] The National Energy Modeling System: An Overview 2009, Tech. rep., Energy Information Administration (EIA) (2009).
URL https://www.eia.gov/outlooks/aeo/nems/overview/pdf/0581(2009).pdf

[25] R. Loulou, G. Goldstein, K. Noble, Documentation for the MARKAL Family of Models, ETSAP (October 2004).
URL https://iea-etsap.org/MrklDoc-I_StdMARKAL.pdf

[26] R. Loulou, U. Remne, A. Kanudia, A. Lehtila, G. Goldstein, Documentation for the TIMES model: Part I, ETSAP (April 2005).

[27] R. Loulou, A. Lehtila, A. Kanudia, U. Remne, G. Goldstein, Documentation for the TIMES model: Part II, ETSAP (April 2005).

[28] N. Strachan, S. Pye, R. Kannan, The iterative contribution and relevance of modelling to uk energy policy, Energy Policy 37 (3) (2009) 850 – 860 (2009). doi:https://doi.org/10.1016/j.enpol.2008.09.096.
URL http://www.sciencedirect.com/science/article/pii/S0301421508005648

[29] H. E. Daly, B. Fais, UK TIMES model overview, Tech. rep., UCL Energy Institute (2014).
URL http://www.ucl.ac.uk/energy-models/models/uktm-ucl/uktm-documentation-overview

[30] International Institute for Applied Systems Analysis (IIASA), MESSAGE model (2013).
URL http://www.iiasa.ac.at/web/home/research/modelsData/MESSAGE/MESSAGE.en.html

[31] W. Short, P. Sullivan, T. Mai, M. Mowers, C. Uriarte, N. Blair, D. Heimiller, A. Martinez, Regional Energy Deployment System (ReEDS), Tech. rep., National Renewable Energy Laboratory (NREL), nREL/TP-6A20-46534 (2011).
URL http://www.nrel.gov/docs/fy12osti/46534.pdf

[32] P. Nahmmacher, E. Schmid, B. Knopf, Documentation of LIMES-EU - a long-term electricity system model for Europe, Potsdam Institute of Climate Impact Research (PIK), https://www.pik-potsdam.de/members/paulnah/limes-eu-documentation-2014.pdf (2014).

[33] T. Mai, E. Drury, K. Eurek, N. Bodington, A. Lopez, A. Perry, Resource Planning Model: An Integrated Resource Planning and Dispatch Tool for Regional Electric Systems, Tech. rep., National Renewable Energy

Laboratory (NREL), nREL/TP-6A20- 56723 (2013).
URL http://www.nrel.gov/docs/fy13osti/56723.pdf

[34] A. Ehrenmann, Y. Smeers, Stochastic Equilibrium Models for Generation Capacity Expansion, Springer New York, New York, NY, 2011, pp. 273–310 (2011). doi:10.1007/978-1-4419-9586-5_13.
URL http://dx.doi.org/10.1007/978-1-4419-9586-5_13

[35] A. Ehrenmann, Y. Smeers, Risk adjusted discounted cash flows in capacity expansion models, Mathematical Programming 140 (2) (2013) 267–293 (2013). doi:10.1007/s10107-013-0692-6.
URL http://dx.doi.org/10.1007/s10107-013-0692-6

[36] F. Murphy, A. Pierru, Y. Smeers, A tutorial on building policy models as mixed-complementarity problems, Interfaces 46 (6) (2016) 465–481 (2016). doi:10.1287/inte.2016.0842.

[37] C. Ruiz, A. J. Conejo, J. D. Fuller, S. A. Gabriel, B. F. Hobbs, A tutorial review of complementarity models for decision-making in energy markets, EURO Journal on Decision Processes 2 (1) (2014) 91–120 (2014). doi:10.1007/s40070-013-0019-0.
URL http://dx.doi.org/10.1007/s40070-013-0019-0

[38] G. Gürkan, O. Ozdemir, Y. Smeers, Generation capacity investments in electricity markets: Perfect competition, CentER Discussion Paper; Vol. 2013-045. Tilburg Econometrics. Available at https://pure.uvt.nl/portal/files/1529759/2013-045.pdf (2013).

[39] K. Poncelet, E. Delarue, D. Six, J. Duerinck, W. D'haeseleer, Impact of the level of temporal and operational detail in energy-system planning models, Applied Energy 162 (2016) 631–643 (Jan. 2016). doi:10.1016/j.apenergy.2015.10.100.

[40] S. Pfenninger, A. Hawkes, J. Keirstead, Energy systems modeling for twenty-first century energy challenges, Renewable and Sustainable Energy Reviews 33 (2014) 74–86 (may 2014). doi:10.1016/j.rser.2014.02.003.

[41] R. H. Koppelaar, J. Keirstead, N. Shah, J. Woods, A review of policy analysis purpose and capabilities of electricity system models, Renewable and Sustainable Energy Reviews 59 (2016) 1531 – 1544 (2016). doi:http://dx.doi.org/10.1016/j.rser.2016.01.090.
URL http://www.sciencedirect.com/science/article/pii/S1364032116001209

[42] H. J. Greenberg, F. H. Murphy, Computing market equilibria with price regulations using mathematical programming, Operations Research 33 (5) (1985) 935–954 (1985). arXiv:https://doi.org/10.1287/opre.33.5.935, doi:10.1287/opre.33.5.935.
URL https://doi.org/10.1287/opre.33.5.935

[43] B. Götz, M. Blesl, U. Fahl, A. Voß, Report on Work Package B-1 of the ETSAP Project "Integrating policy instruments into the TIMES Model": The explicit modeling of support systems for renewable electricity in TIMES, Tech. rep., University of Stuttgart (2012).
URL https://iea-etsap.org/projects/ETSAP_PI_WPB1.pdf

[44] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, C. Ruiz, Complementarity Modeling in Energy Markets, Springer-Verlag New York, 2013 (2013). doi:10.1007/978-1-4419-6123-5.

[45] J. Zhao, B. F. Hobbs, J.-S. Pang, Long-run equilibrium modeling of emissions allowance allocation systems in electric power markets, Operations Research 58 (3) (2010) 529–548 (2010). doi:10.1287/opre.1090.0771.

[46] A. Ehrenmann, Y. Smeers, Generation capacity expansion in a risky environment: A stochastic equilibrium analysis, Operations Research 59 (6) (2011) 1332–1346 (2011). doi:10.1287/opre.1110.0992.

[47] G. Oggioni, Y. Smeers, Evaluating the impact of average cost based contracts on the industrial sector in the european emission trading scheme, Central European Journal of Operations Research 17 (2) (2009) 181–217 (2009). doi:10.1007/s10100-008-0083-x.
URL http://dx.doi.org/10.1007/s10100-008-0083-x

[48] J. Bushnell, J. Ishii, An equilibrium model of investment in restructured electricity markets, center for the Study of Energy Markets Working Paper 164. UC Berkeley. Available at http://escholarship.org/uc/item/5x5543dg (2007).

[49] A. Schröder, T. Traber, C. Kemfert, Market driven power plant investment perspectives in europe: Climate policy and technology scenarios until 2050 in the model emelie-esy, Climate Change Economics 04 (supp01) (2013) 1340007 (2013). arXiv:http://www.worldscientific.com/doi/pdf/10.1142/S2010007813400071, doi:10.1142/S2010007813400071.
URL http://www.worldscientific.com/doi/abs/10.1142/S2010007813400071

[50] S. Wogrin, J. Barquín, E. Centeno, Capacity expansion equilibria in liberalized electricity markets: An epec approach, IEEE Transactions on Power Systems 28 (2) (2013) 1531–1539 (May 2013). doi:10.1109/TPWRS.2012.2217510.

[51] X. Hu, D. Ralph, Using EPECs to Model Bilevel Games in Restructured Electricity Markets with Locational Prices, Operations Research 55 (5) (2007) 809–827 (2007). doi:10.1287/opre.1070.0431.

[52] L. P. Garces, A. J. Conejo, R. Garcia-Bertrand, R. Romero, A bilevel approach to transmission expansion planning within a market environment, IEEE Transactions on Power Systems 24 (3) (2009) 1513–1522 (Aug 2009). doi:10.1109/TPWRS.2009.2021230.

[53] A. J. Conejo, E. Castillo, R. Minguez, R. Garcia-Bertrand, Decomposition Techniques in Mathematical Programming, Springer-Verlag Berlin Heidelberg, 2006 (2006).

[54] B. Tennbakk, A. von Schemde, D. Six, Electricity Market Functioning: Current Distortions, and How to Model Their Removal - FINAL REPORT, Tech. rep. (2016).
URL http://ec.europa.eu/energy/en/studies/electricity-market-functioning-current-distortions-and-how-model

[55] ELIA, Grid data - Elia (Apr. 2015).
URL http://www.elia.be/en/grid-data

[56] K. Poncelet, H. Höschle, E. Delarue, A. Virag, W. D'haeseleer, Selecting representative days for capturing the implications of integrating intermittent renewables in generation expansion planning problems, IEEE Transactions on Power Systems 32 (3) (2017) 1936–1948 (May 2017). doi:10.1109/TPWRS.2016.2596803.

[57] F. Olsina, F. Garcés, H.-J. Haubrich, Modeling long-term dynamics of electricity markets, Energy Policy 34 (12) (2006) 1411 – 1433 (2006). doi:http://doi.org/10.1016/j.enpol.2004.11.003.
URL http://www.sciencedirect.com/science/article/pii/S0301421504003325

[58] D. Ralph, Y. Smeers, Pricing risk under risk measures: An introduction to stochastic-endogenous equilibria, available at SSRN: http://escholarship.org/uc/item/5x5543dg (2011).