Soft tissue artefact compensation by L-curved Tikhonov regularization filtering in motion analysis

W. Kim¹, A. Veloso¹, J. Tan², Vera Moniz¹, Silvia Cabral¹
¹Faculty of Human Kinetics, Technical University of Lisbon, Estrada da Costa, Cruz-Quebrada, Portugal
²Physical Education & Sports Science, National Institute of Education, Singapore 639798

Abstract
Researchers have reported on several compensation methods to estimate bone pose from a cluster of skin-mounted markers, each influenced by soft tissue artefact (STA)—i.e. for the determination of rigid body transformation parameters from ill-conditioned spatial marker coordinates. This study aimed to estimate the knee joint motion during the gait, using a purposely developed STA compensation tool/method consisting of an L-curved Tikhonov regularization filtering (LTRF) algorithm. The LTRF, which was validated via processing of spatial-temporal estimates of knee joint motion, incorporated a novel use of L-curve approach. Investigation of the effectiveness of the regularization parameters, using said algorithm, reveals the optimal parameters as $3.1623 \times 10^{-12}$ for the markers located on shank. The LTRF approach implemented by varying the smoothing parameter may be widely applicable to the necessary process of combined filtering and STA compensation.

Introduction
The strategies of minimizing modelling errors, as a means of compensation of STA, is the one that has received much attention from both researchers and practitioners (Leardini, Chiari et al. 2005). Techniques designed to minimize the contribution of and compensation for STA can be divided into those which model the skin surface and those which include joint motion constraints. A “solidification” procedure was proposed on the assumption that markers’ trajectories are consistent with the rigid body motion (Cheze, Fregly et al. 1995). A different approach is the principal axes of inertial method in which each segment marker is assigned a mass that can act as a probability density function (PDF) or weighting factor (Andriacchi, Alexander et al. 1998). The centre of mass and the inertial tensor of the cluster are calculated at each time frame based on the analogy that the inertial tensor about the centre of mass of a 3-D rigid body is related to the covariance matrix of trivariate random vectors, whose PDF is proportional to the point wise density of the rigid body itself. The global optimization treats each body segment in holistic terms – i.e. for the structure that is undergoing transformation as a whole –rather than in terms of a separate segment, with imposition constrains at the joints (Lu and O’Connor 1999). This process was defined by minimizing the weighted sums of squares distance between simulation and model determined marker positions. All the above compensation techniques have common features: The transformation parameter (both the rotation and translation) can be computed from the linear interpolation as for affine mapping; subsequently the least-squared estimates (LSE) were adopted by means of extracting transformation parameters between two point patterns via SVD (Singular Value Decomposition). However, this LSE is not sufficient, because a solution that will minimize error will end up with the model exactly matching the data—a situation that is to be avoid for STA compensation. This is where the regularization method (Tikhonov, 1997) enters. In this study, therefore, we estimated spatial-temporal knee joint motion, by incorporating a novel use of STA compensation using the L-curved Tikhonov regularization filtering (LTRF) algorithm.

Method
We will use a third-order model that will allow us to estimate the first and second derivatives of data. The model is

$$\dot{x} = v$$
$$\dot{v} = a$$ (1)
$$\dot{a} = \dot{g}$$

The model is to be driven by the forcing function $g(t)$, and the discrete model becomes

$$z_{j+1} = \begin{bmatrix} x_{j+1} \\ v_{j+1} \\ a_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & h & h^2 / 2 \\ 0 & 1 & h \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_j \\ v_j \\ a_j \end{bmatrix} + \begin{bmatrix} h^3 / 6 \\ h^2 / 2 \\ h \end{bmatrix} \begin{bmatrix} g_j \end{bmatrix}$$ (2)

where $h$ represents a sampling time step. The error term is a combination of matching the data $d_j$ and the regularization of $g_j$. The least square error term is now expressed as

$$E_N(c, g_j) = \sum_{j=1}^{N} (d_j - x_j)^2 + bg_j^2$$ (3)

What is now required of the solution is to best match the data (the first term of Equation 3), but also to have some degree of smoothness (the second term). By adding a term to the LSE error function in Equation 3, one can control the amount of smoothness that occurs in the solution by varying the parameter $b$. The smoothing problem is to find the forcing term $g_j$ and the initial condition $c$ that minimizing the error function $E_N$ and to also determine the optimal values of the smoothing parameter $b$. The addition of the term $bg_j^2$ is crucial to obtaining smooth and reasonable results. It is known as the regularization term and is sometimes referred to as Tikhonov method. There is a method that can be used to estimate the optimal value of $b$. It is called generalized cross validation or L-curve (Trujillo and Busby, 1997). In essence, it is desired to find the input force ($g_j$) that causes the model ($x_j$) to match the data ($d_j$) as closely as possible. Thus, the problem is to minimize the function $E_N$ over the sequence of forcing vector $g_j$. This can be solved using dynamic programming and Bellman’s Principle of Optimality (Trujillo, 1997). This leads to

$$f_N(c) = \min_{g_j} E_N(c, g_j)$$ (4)

A recurrence formula can be derived by applying the principle of optimality, which yields
The recursion formula can be found in previous works from the first authors (Kim 2009).

**Results**

We present results applying our LTRF approach to published datasets produced from stance phase of level walking (Schipplein 1991). The experimental gait data was collected from an adult male subject implanted with an instrumented knee replacement and analyzed using LTRF method. Coordinates acquired from shank and thigh makers were used to generate relative instantaneous screw axis (ISA) and synchronized to GRFs with associated COP data obtained during two gait cycles. We have generated the ISA using a plucker Coordinate-technique (Kiat 2006). The ISA was linked to the GRF, by coupling both screws, and plot in the spatiotemporal migration (Figure 1). Upon-processing marker coordinates from the shank, the LTRF indicated a distinct ‘corner’ which was associated with the optimized values of the regularization parameter. Thus, the flexion point on the logarithmic plot of the iterative solution versus the residuals was chosen as $b = 3.1623 \times 10^{-12}$ (Figure 2).

![Figure 1: In a validation effort compensating STA in time-sequence of motion data, ISA screws are shown to regularly intersect the GRF wrenches as indicated with the colored lines progressing in time and space (X-direction). This representative analysis indicates a reciprocal connection (Spatial-temporal representation of knee motion versus the foot loading) between the motion of the knee joint and the forces applied to the joint via the plantar reactions.](image)

![Figure 2: The Tikhonov L-curve for this particular set of makers on the shank. A distinct ‘corner’ in the plot was associated with the regularization parameter $b$ scalar, optimized through analytical iteration. The L-curve displayed information about the regularized solution by plotting the iterative solution of Equation (3) versus the residual vectors (the deviation between observed and predicted values) typically on logarithmic axes. In the simplified application here, a scalar of the regularization parameter $b$ was chosen by association with the characteristic reflection point on the plot.](image)

References


